Preparation for ECON-6570 Advanced Econometrics

Ken Simons, version 10 Dec. 2010

In preparation for the Advanced Econometrics course, please review fundamental skills with key topics used in the course: matrix algebra, exponents and logarithms, calculus, and probability. To review most of these topics, you could use any standard textbook on quantitative methods for economics, or appendices A and B of William H. Greene's book *Econometric Analysis* (6th edition). Most importantly, though, look over the review materials I am providing for you. These are important to ensure that everyone has some basic background at the start of term.

Details are below.

Matrix Algebra

We will use matrix algebra throughout our PhD Econometrics course. Many of you know all of this already, but some people will need some review. If you don't know part or all of the following, don't panic, but do look it up ASAP. You can find some introductions on the internet (e.g., http://www.sosmath.com/matrix/matrix.html), or try a mathematics-for-economics book like Alpha Chiang's *Fundamental Methods of Mathematical Economics*.

You should know:

- What is a matrix
- How to add and subtract matrices
- How to multiply a matrix by a constant and by another matrix
- How to transpose a matrix
- What are the following types of matrices: square, symmetric, diagonal, identity, upper triangular, lower triangular, idempotent
- What is an inverse of a matrix (don't worry about how to compute it)
- What is the rank of a matrix, how to compute the determinant of a 2x2 matrix, and what you know about a matrix's rank if its determinant equals zero

Using my matrix algebra practice sheets (see below), you should also learn how to compute derivatives and solve optimization problems in matrix form. For optimization, the practice sheets discuss unconstrained optimization, but it would help to understand equality-constrained optimization also (see a mathematics-for-economics book).

Exponents and Logarithms and Calculus

In addition to matrix algebra, you should have some basic skills for exponents and logarithms and for differentiation. If you are forgetting, please look these up. You should be able to simplify formulae involving exponents and logarithms, and you should be able to sketch the graphs of exponents and logarithms. You should understand what a derivative means, you should be able to compute derivatives for common formulae

including fractions, multiples, logarithms, and exponents; and you should be able to use the chain rule for derivatives. Also, you should know what is an integral and how to write (definite) integrals in formulae. (Solving integrals using formulas is of minor importance for this course but it would help to learn to solve a few basic integral formulas.)

Probability

Finally, the course will assume some basic knowledge of probability. You should understand probability density functions, for one or multiple random variables, and the associated cumulative density functions. You should understand and know how to compute expectations including mean and variance, as well as conditional expectations. You should understand joint probability density functions and how to compute conditional probability density functions. For all of these you should be able to work with both discrete and continuous random variables. If you have never learned these or need review, you could use almost any standard book on the topic. Personally I like Alvin W. Drake's old textbook *Fundamentals of Applied Probability Theory* (McGraw-Hill, 1967), which is dense but clear (see chapter 1 and especially chapter 2, and practice by covering up the solutions to the examples and solving them yourself).

A background in statistics is not assumed, but would help. You could look at any standard textbook on statistics, or Greene's appendices C-D in *Econometric Analysis* (6th edition). I will summarize but not prove key statistical theorems used in class.

Practice Sheets

I have prepared practice sheets with solutions, for much of the above: matrix algebra including quadratic forms, exponents and logarithms, and derivatives. You will definitely need to do the matrix algebra practice sheets; please complete this before our first meeting (especially if you are weak on matrix algebra). The exponents & logarithms and derivatives practice sheets are useful *if* you need to review those topics, and the derivatives sheets are lengthier than we really need for this course (I use the derivatives sheets for several courses).

Some Key Facts

The matrix algebra practice sheets point out the following facts, which you should learn by heart for this course. For any matrices $\mathbf{U}$ and $\mathbf{V}$ for which the relevant additions, multiplications, or inverses can be carried out:

$$(\mathbf{U}+\mathbf{V})' = \mathbf{U}' + \mathbf{V}'$$

$$(\mathbf{UV})' = \mathbf{V}'\mathbf{U}'$$

$$\mathbf{U}^{-1}\mathbf{U} = \mathbf{U}\mathbf{U}^{-1} = \mathbf{I}$$

$$(\mathbf{UV})^{-1} = \mathbf{V}^{-1}\mathbf{U}^{-1}$$

The matrix algebra sheets will also familiarize you with the following facts, which we will use during the course and are worth memorizing:

$$\frac{\partial(\mathbf{a}'\mathbf{b})}{\partial \mathbf{b}} = \mathbf{a}$$

$$\frac{\partial(\mathbf{b}'\mathbf{A}\mathbf{b})}{\partial \mathbf{b}} = 2\mathbf{A}\mathbf{b} \text{ if } \mathbf{A} \text{ is symmetric}$$

A scalar $f(\mathbf{b})$ is minimized with respect to a K-dimensional vector $\mathbf{b}$ if

$$\frac{\partial f(\mathbf{b})}{\partial \mathbf{b}} = \mathbf{0} \text{ and } \frac{\partial^2 f(\mathbf{b})}{\partial \mathbf{b} \partial \mathbf{b}'} \text{ is positive definite (i.e. } \mathbf{x}' \frac{\partial^2 f(\mathbf{b})}{\partial \mathbf{b} \partial \mathbf{b}'} \mathbf{x} > 0$$

for all K-dimensional vectors $\mathbf{x} \neq \mathbf{0}$)

The following facts from probability will be used often; it will help to understand these facts and why these are true:

$\mathrm{E}[f(\mathbf{x})y \mid \mathbf{x}] = f(\mathbf{x})\mathrm{E}(y \mid \mathbf{x})$ "linearity of conditional expectations"

$\mathrm{E}[\mathrm{E}(y \mid \mathbf{x})] = \mathrm{E}(y)$ "Law of Total Expectations"

$\mathrm{E}[\mathrm{E}(y \mid \mathbf{x}, \mathbf{z}) \mid \mathbf{x}] = \mathrm{E}(y \mid \mathbf{x})$ "Law of Iterated Expectations"

Try writing these out as integrals using the definition of an expectation or conditional expectation, then solving, so that you know why these are true. Then make sure you remember these facts and their names.