Answers to Practice with Derivatives

1) 0
2) 3
3) $7x^6$
4) $35x^6$
5) $35x^6 + \frac{1}{2}x^{-\frac{1}{2}} - 3x^{-4}$
6) e^x
7) $2e^{2x}$
8) $10e^{2x}$
9) ae^{ax}
10) $2nx^{n-1} - ae^{ax}$ (or $-ae^{ax}$ if $n = 0$)
11) $(35x^6)(e^x) + (5x^7)(e^x) = 5(x^7 + 7x^6)e^x$
12) $2e^{2x}x^{-\frac{3}{2}} - \frac{3}{2}e^{2x}x^{-\frac{5}{2}} = \left(2x^{-\frac{3}{2}} - \frac{3}{2}x^{-\frac{5}{2}}\right)e^{2x}$
13) $(10x + 94x^{93})(e^{2x} + 17x + 2) + (5x^2 + x^{94} + 12)(2e^{2x} + 17)$
14) Either use the rule for fractions, or write the original expression as $x(x+2)^{-1}$ and use the rule for multiples. The answer is: $\frac{1}{x+2} - \frac{x}{(x+2)^2} = \frac{x+2-x}{(x+2)^2} = \frac{2}{(x+2)^2}$
15) $\frac{7x^6}{x^2+1} - \frac{x^7}{(x^2+1)^2}2x = \frac{7x^6(x^2+1) - 2x^8}{(x^2+1)^2} = \frac{7x^6 + 5x^8}{(1+x^2)^2}$
16) $\left(\frac{x^7}{x^2+1}\right)3e^{3x} + \frac{7x^6 + 5x^8}{(1+x^2)^2}\left(e^{3x} + 24\right)$
17) $f'(x) \cdot g(x) + f(x) \cdot g'(x)$
18) $\frac{f'(x)}{g(x)} - \frac{f(x)}{g(x)^2}g'(x) + 17 = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2} + 17$
19) Use the rule for multiples twice with $f(x) \cdot [g(x) \cdot h(x)]$, with the result $f'(x) \cdot g(x) \cdot h(x) + f(x) \cdot g'(x) \cdot h(x) + f(x) \cdot g(x) \cdot h'(x)$
20) Use the chain rule to obtain $e^{x^2}(2x) = 2xe^{x^2}$
21) Use the chain rule to obtain $32(e^x + 17x)^{31}(e^x + 17)$
22) $(2x + 37)\exp(x^2 + 37x + 3)$

23) $f'(g(x))g'(x)$

24) $32\left(e^{x^2}+17x\right)^{31}\left(2xe^{x^2}+17\right)$

25) $f'(g(h(x)))g'(h(x))h'(x)$

26) $\cos(x)$

27) $6\cos(2x)$

28) $-\sin(x)$

29) $-6\sin(\frac{1}{2}x)$

30) $5^x\ln(5)$

31) $-\sin(5^x+7)5^x\ln(5)$

32) $\cos(3+7e^x+\cos(5^x+7+e^{2x}))\left(7e^x-\sin(5^x+7+e^{2x})\left(5^x\ln 5+2e^{2x}\right)\right)$

33) $\cos(\sin(\sin(\sin(x))))\cdot\cos(\sin(\sin(x)))\cdot\cos(\sin(x))\cdot\cos(x)$

34) $\exp(\exp(\exp(\exp(x))))\cdot\exp(\exp(\exp(x)))\cdot\exp(\exp(x))\cdot\exp(x)$

35) The original expression is almost all constants: it is $A(B-17x)^{-1}$, where $A=\pi\sqrt{84}-\sin(17)+9734k-\frac{284}{\pi\sqrt{3}}$ and $B=184\left(\frac{\pi}{\sqrt{3}}+84-\sin(12)\right)$. The answer is: $17A(B-17x)^{-2}$

36) $3\cos\left(\pi-284e^{x^2}-\frac{\sqrt{2976}\cos(84)}{3\sqrt{7}-5\sqrt{2}}\right)\left(-284e^{x^2}\cdot 2x\right)$

37) $\frac{1}{x}$

38) $\frac{3}{\frac{5}{2}+8x^2}\cdot 16x=\frac{96x}{5+16x^2}$

For questions 39-49, let u be the original expression. Then the answers are as follows:

39) $\frac{\partial u}{\partial x}=7y$, $\frac{\partial u}{\partial y}=7x$

40) $\frac{\partial u}{\partial x}=2$, $\frac{\partial u}{\partial y}=21$

41) $\frac{\partial u}{\partial x}=e^x$, $\frac{\partial u}{\partial y}=-7$

42) $\frac{\partial u}{\partial x} = 4xye^{2x^2y} - y$, $\frac{\partial u}{\partial y} = 2x^2e^{2x^2y} - x$

43) $\frac{\partial u}{\partial x} = y \cdot \cos(\cos(\sin(\cos(xy)))) \cdot \sin(\sin(\cos(xy))) \cdot \cos(\cos(xy)) \cdot \sin(xy)$,

$\frac{\partial u}{\partial y} = x \cdot \cos(\cos(\sin(\cos(xy)))) \cdot \sin(\sin(\cos(xy))) \cdot \cos(\cos(xy)) \cdot \sin(xy)$

44) Before solving, it is possible to note that $\sqrt{\pi x^2} = x\sqrt{\pi}$.

$\frac{\partial u}{\partial x} = \frac{512\sqrt{\pi}}{21y^{84}}$, $\frac{\partial u}{\partial y} = -84 \cdot \frac{512\sqrt{\pi}x}{21y^{85}}$

45) $\frac{\partial u}{\partial x} = f'(x)$, $\frac{\partial u}{\partial y} = g'(y)$

46) $\frac{\partial u}{\partial x} = \frac{\partial f(x,y)}{\partial x} \cdot g(x,y) + f(x,y) \cdot \frac{\partial g(x,y)}{\partial x}$,

$\frac{\partial u}{\partial y} = \frac{\partial f(x,y)}{\partial y} \cdot g(x,y) + f(x,y) \cdot \frac{\partial g(x,y)}{\partial y}$

47) $\frac{\partial u}{\partial x} = yx^{y-1}$, $\frac{\partial u}{\partial y} = x^y \ln x$ (This is well-defined for $x > 0$, but not for many other combinations of x and y.)

48) $\frac{\partial u}{\partial x} = 2yx^{2y-1}$, $\frac{\partial u}{\partial y} = x^{2y} \ln x$ (This is well-defined for $x > 0$, but not for many other combinations of x and y.)

49) $$\frac{\partial u}{\partial x} = \frac{\left(xy + xy^2 + x^2y + \sin(xy)\right) - \left(x - 3\sqrt{y} + 84y^2 - 7\right)\left(y + y^2 + 2xy + y\cos(xy)\right)}{\left(xy + xy^2 + x^2y + \sin(xy)\right)^2}$$

,

$$\frac{\partial u}{\partial y} = \frac{\left(xy + xy^2 + x^2y + \sin(xy)\right)\left(-\frac{3}{2}y^{-3/2} + 168y\right) - \left(x - 3\sqrt{y} + 84y^2 - 7\right)\left(x + 2xy + x^2 + x\cos(xy)\right)}{\left(xy + xy^2 + x^2y + \sin(xy)\right)^2}$$

Often people simplify, by writing expressions with variables that actually equal more complex expressions. Here, x and y are functions of t, suggesting that the writer is thinking of functions of time. When many variables are functions of time, it is common to drop the "(t)", as in expressions 50-52. The time- (or other-) derivative of a variable often is written simply by placing a dot over the variable name, for example, $\dot{x} = \frac{dx(t)}{dt}$ and $\dot{y} = \frac{dy(t)}{dt}$.

50) $\dot{x} + \dot{y}$

51) $2x\dot{x}y^2 + 2x^2y\dot{y}$

52) $\dfrac{A-B}{C^2}$, where $A=\left(\dot{x}-\dfrac{3}{2}y^{-\frac{1}{2}}\dot{y}+168y\dot{y}\right)C$,

$B=\left(x-3\sqrt{y}+84y^2-7\right)\left(\dot{x}y+x\dot{y}+\dot{x}y^2+2xy\dot{y}+2x\dot{x}y+x^2\dot{y}+\cos(xy)\left(\dot{x}y+x\dot{y}\right)\right)$

, and $C=xy+xy^2+x^2y+\sin(xy)$

The total differential of u(x,y) is $du=\dfrac{\partial u}{\partial x}dx+\dfrac{\partial u}{\partial y}dy$. Effectively, this tells the infinitesimal change in u that would result from infinitesimal changes in both x and y.

53) $dx+dy$

54) $2xy^2dx+2x^2ydy$

55) Use the answer to problem 49 to get $\dfrac{\partial u}{\partial x}dx+\dfrac{\partial u}{\partial y}dy=$

$$\frac{\left(xy+xy^2+x^2y+\sin(xy)\right)-\left(x-3\sqrt{y}+84y^2-7\right)\left(y+y^2+2xy+y\cos(xy)\right)}{\left(xy+xy^2+x^2y+\sin(xy)\right)^2}dx$$
$$+\frac{\left(xy+xy^2+x^2y+\sin(xy)\right)\left(-3y^{-3/2}+168y\right)-\left(x-3\sqrt{y}+84y^2-7\right)\left(x+2xy+x^2+x\cos(xy)\right)}{\left(xy+xy^2+x^2y+\sin(xy)\right)^2}dy$$

Answers are not shown for questions 56-61. Make sure you understand how the numbers in the graph of $y'(x)$ tell the slope at each point in the graph of $y(x)$.

56)
57)
58)
59)
60)
61)

For questions 62-65, first compute the derivative, $\dfrac{d f(x)}{dx}$. Then, using the resulting expression, take the derivative again to get $\dfrac{d^2 f(x)}{dx^2}=\dfrac{d}{dx}\left(\dfrac{d f(x)}{dx}\right)$. Continue for the third and fourth derivatives.

62) $\dfrac{d f(x)}{dx}=3$, so $\dfrac{d^2 f(x)}{dx^2}=0$, $\dfrac{d^3 f(x)}{dx^3}=0$, $\dfrac{d^4 f(x)}{dx^4}=0$

63) $\frac{d f(x)}{dx} = 35x^6 + \frac{1}{2}x^{-\frac{1}{2}} - 3x^{-4}$, so $\frac{d^2 f(x)}{dx^2} = 210x^5 - \frac{1}{4}x^{-\frac{3}{2}} + 12x^{-5}$,
$\frac{d^3 f(x)}{dx^3} = 1050x^4 + \frac{3}{8}x^{-\frac{5}{2}} - 60x^{-6}$, $\frac{d^4 f(x)}{dx^4} = 4200x^3 - \frac{15}{16}x^{-\frac{7}{2}} + 360x^{-7}$

64) $\frac{d f(x)}{dx} = ae^{ax}$, so $\frac{d^2 f(x)}{dx^2} = a^2 e^{ax}$, $\frac{d^3 f(x)}{dx^3} = a^3 e^{ax}$, $\frac{d^4 f(x)}{dx^4} = a^4 e^{ax}$

65) $\frac{d f(x)}{dx} = -3\sin(3x)$, so $\frac{d^2 f(x)}{dx^2} = -9\cos(3x)$, $\frac{d^3 f(x)}{dx^3} = 27\sin(3x)$,
$\frac{d^4 f(x)}{dx^4} = 81\cos(3x)$