Fractals, Complexity, and Connectivity in Africa

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Part 1. Introduction

The question "What mathematics from Africa?" can be seen as a conflation of two pertinent, related questions. First, "What mathematics?" Second, "What Africa?" Answering these will enable us to more fully explore the subject of our inquiry.

The first question "What mathematics?" is of course nonsensical as a statement. What we are really trying to ask, is "What do we mean by mathematics?". Is our mathematics a verb or a noun? It is important to ask this question as there are many different understandings of what mathematics actually is, and controversy surrounds any effort to try and define mathematics. A comprehensive look at the question of the nature of mathematics is beyond the scope of this paper, and we will consciously delimit our discussion of mathematics to mathematics as *practice* – by which we refer to systematic material and symbolic understandings of quantity and logic. A simple example of what we mean by mathematics as practice can be seen in architecture. For example, much of New York City is laid out as a grid, which may be seen as a collection of rectangles. It also includes an ordinal dimension in that streets can be numbered. For the inhabitants of a built environment, the very obviousness of certain patterns can make them invisible. We don't realize that we are surrounded by the shapes of Euclid – circles, rectangles and triangles, etc. – because we don't have built fractal structures to contrast them with. As we will see, fractal geometry not only illuminates the underlying structure of African designs, but also helps us see the cultural-boundedness of our own mathematical practice.

Most people engage in mathematical exercise unknowingly everyday. Determining change in financial transactions is just one of the many abilities that require mathematical sensibilities. Time and space are also mathematically delimited as we all refer to our clocks for local time and unconsciously resolve the mathematical relationship between seconds, minutes and hours. The argument can easily be made that mathematics is all around us. This point must be fully understood as we proceed to discuss mathematics from Africa.

The second question, "What Africa?" alas may not be as easily resolvable. In our opinion, the term Africa is used as a placeholder for all things relating to that particular continent. For example, we often hear phrases similar to "she visited Brazil, Mexico, London and then Africa." – thus conflating the continent of Africa with countries (Brazil and Mexico). The continent of Africa is the second largest in the world and is geopolitically comprised of many countries, cities and people groups. The critical position that we must strive to recover is that of *difference* (Appiah 1992; Eglash 1999). Africa is not homogenous; either culturally, or politically, or even architecturally. It could be convincingly argued that there is no singular "African" identity (though some would add that one is evolving). Similarly, to speak about an "African" mathematics is to oversimplify a very complex phenomena. While our discussions oscillate around an "African mathematics" we must emphasize that at best, we are describing a subtle family resemblance across multiple cultural streams.

Part 2. Previous work on African Fractals

In this section we review the previous work on fractals in African traditional and hybrid knowledge systems. This is explained in detail in the book **African Fractals**; here we provide only a brief overview.

Users new to fractal geometry can think of fractals simply as patterns that repeat themselves at many different scales. Fractal geometry was first applied in studies of natural systems: trees are branches of branches, mountains are peaks of peaks, and clouds are puffs of puffs. That is because nature tends to utilize self-organizing processes in its constructions, such as clusters of cells forming clusters of clusters. Eglash (1989) first noted fractal structures in aerial photos of African villages. This visual observation has been confirmed through simulation (see illustrations below), and through measure of the fractal dimension of the images (using both box-counting and spectral density function methods). The initial hypothesis was that these structures, like those in nature, were created by unconscious self-organization, but a year of fieldwork in West and Central Africa involving interviews with the artisans provided convincing evidence that they are better classified as intentional, conscious designs: that something analogous to fractal construction techniques – recursive scaling practices – exists as an African indigenous knowledge system.

One of the barriers to understanding this is the Western concept of intentionality. In the Euro-American tradition an intentional design is one which has someone in charge, and gets completed on a deadline. Occasionally these criteria are violated, but when they are violated the status of the design is compromised (cf. examples of cathedral construction in Turnbull 2000). In the African case many villages were constructed over many generations with no one in charge – yet there is a cohesive fractal pattern for the village as a whole.

Figure 1 shows one of many examples. Ring-shaped livestock pens, one for each extended family, can be seen in the aerial photo in Figure 1a, a Ba-ila settlement in southern Zambia. Towards the back of each family ring we find the family living quarters, and towards the front is the gated entrance for letting livestock in and out. For this reason the front entrance is associated with low status (unclean, animals), and the back end with high status (clean, people). This scaling of social status is reflected by the scaling in the architecture of each family ring: the front of the ring is only fencing, as we go towards the back smaller buildings (for storage) appear, and towards the very back end are the larger houses. The two geometric elements of this structure -- a ring shape overall, and a status gradient increasing with size from front to back -- echo throughout every scale of the Ba-ila settlement.

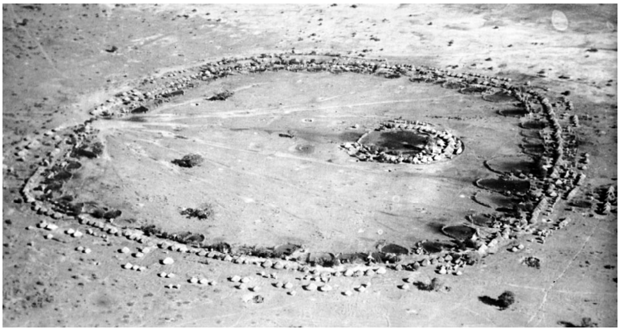


Figure 1a. Aerial view of a Ba-ila settlement in southern Zambia.

At the smallest scale, if we were to view a single house in the family ring from above, we would see that it is a ring with a special place at the back of the interior: the household altar. The settlement as a whole has the same shape: it is a ring of rings. The settlement, like the family ring, has a front/back social distinction: the entrance is low status, and the back end is high status. At the settlement entrance there are no family enclosures at all for the first 20 yards or so, but the farther back we go, the larger the family enclosures become.

At the back end of the interior of the settlement, we see a smaller detached ring of houses, like a settlement within the settlement. This is the chief's extended family ring. At the back of the interior of the chief's extended family ring, the chief has his own house. Enclosed by that ring is an alter composed of miniature houses representing the spiritual presence of ancestors. The scaling sequence allows a kind of infinite regress into the spiritual realm.

Since we have a similar structure at all scales, this architecture is easy to model with fractals. Figure 1b shows the first three iterations. We begin with a seed shape that could be the

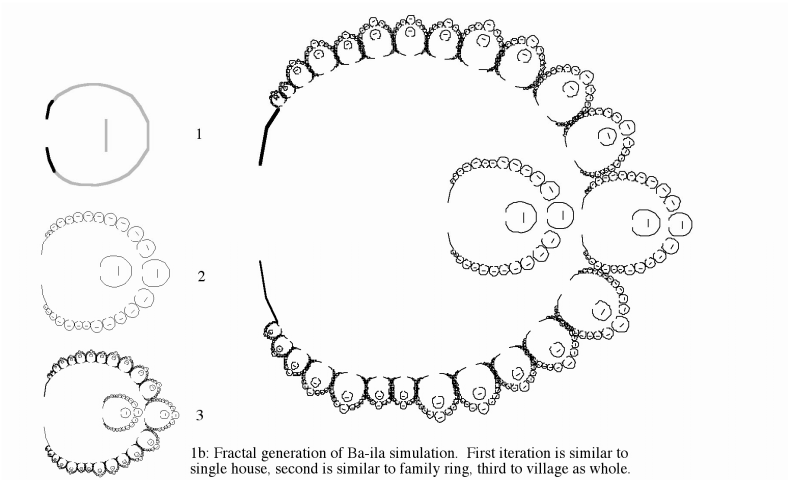


Figure 1b. Fractal pattern in Ba-ila settlement.

overhead view of a single house. In the next iteration, we have a shape that could be the overhead view of a family enclosure. Finally, the third iteration provides a structure that could be the overhead view of the whole settlement. Note that the self-replicating line off-center is the position of the altar in the house, and the position of the village altar in the chief's family ring: a resonance between spiritual notions of endless renewal, mathematical models of self-replication, and physical structure that we see in many of the fractal village architectures of Africa.

Such architectural fractals abound in African village structures, some rectangular rather than circular, some much more diffuse than coherent, but still exhibiting a scaling characteristic. Scaling architectures, however, are only the start. Fractal characteristics can also be seen in African textiles, paintings, sculpture, masks, religious icons, cosmologies, and social structures. Age grade systems, for example, sometimes symbolize iterations in social status with iterations in geometric structures of masks or sand drawings. Religious symbols include recursively nested calabashes, snakes of "infinite length" coiled into a finite space, crosses-within-crosses-within-crosses, and other icons that express cosmologically nested infinities with their geometric recursion.

Even numeric systems in African can have fractal characteristics. Take, for example, the game variously called "ayo," "bao," "giuthi," "lela," "mancala," "omweso," "owari," "tei," and "songo" (among many other names). Players in Ghana use the term "marching group" for a self-replicating pattern, such as the example below. Here the number of counters in a series of cups each decrease by one (e.g. 4-3-2-1). The entire pattern can be replicated with a right-shift by scooping from the largest cup, and placing one counter in each succeeding cup (figure 2). If left uninterrupted, repeated application will allow it to propagate in

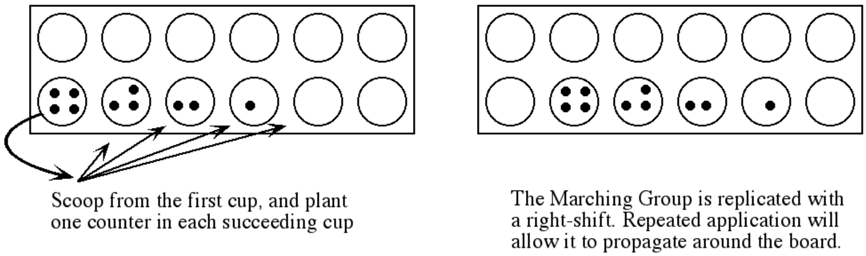
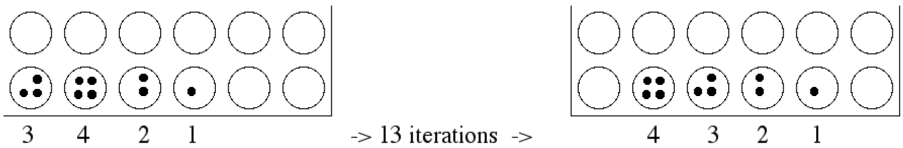


Figure 2. The "marching group" in Owari.

this way as far as needed. As simple as it seems, this concept of a self replicating pattern is at the heart of some sophisticated mathematical concepts. Figure 3 shows how the owari marching group system can be used as a one-dimensional cellular automaton, demonstrating many of the dynamic phenomena produced on two-dimensional systems.[1]



In the example of figure 2 we saw how the marching group can replicate in one iteration. However that was for an initial state which is the same as the final state. In this case we see how a different initial state requires 13 repeated applications of the marching group rule (scoop from the left-most cup and deposit in the succeeding cups) before it can achieve the final state.

This is similar to a "transient" in a one-dimensional cellular automaton, before it settles into its final state. Transients of maximum length are used as a specific tactic by Ghanian players, who call it "slow motion." In nonlinear dynamics terminology, this would be a "point attractor" and the set of all its transients would constitute its "basin of attraction." The marching group rule can also produce periodic behavior (a periodic attractor). For example here is a period 3 system using a total of four counters: 211 → 22 → 31→211

Which total number of counters lead to point attractors, and which ones to periodic attractors?

[1] One dimensional versions can show all the dynamics of two dimensions, and can even be used as a kind of parallel computer. Consider, for example, a rule that in each iteration the number of counters in a cup is replaced by the sum of itself and its left neighbor. Starting with one: 0100000 -> 0110000 -> 0121000 -> 0133100 -> 0146410. This fourth iteration gives us the binomial coefficients for expansion of $(a + b)^4$, which equals $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$.

Total number of counters	Behavior (after transients)
1	Marching
2	Period 2
3	Marching
4	Period 3
5	Period 3
6	Marching
7	Period 4
8	Period 4
9	Period 4
10	Marching
11	Period 5
12	Period 5
13	Period 5
14	Period 5
15	Marching

Figure 3. Owari as a one-dimensional cellular automaton

Here we can see that the number of counters that lead to marching groups are 1,3,6,10,15… – the triangle number sequence. This sequence is a common signature of recursive or self-organizing systems, and is featured in another African game, "tarumbeta".

Is it possible to create a chaotic attractor using Owari? We invite the readers to attempt to develop a modification to the marching group rule that leads to true deterministic chaos.

Part 3. African Mathematics as Synecdoche

The valid question arises, in what ways can an understanding of African mathematical representations, fractals, complexity and chaos in indigenous cultural practices assist us in theorizing about the future? One immediate pitfall that must be avoided at all costs is the desire to look to the past for answers to the present. The discussion of African mathematical representations must not be interpreted as a desire for historical ascendancy rooted in myths of some pure past. It cannot be denied that the continent is facing a number of challenges now and ahead, however, we do not believe that the answers lie in the past. What is needed is rather a hybridization of the past within the present to help envision a better future.

In our opinion, probably, the greatest point that can be taken away from our recent research Eglash (1999) into African mathematics is an appreciation of African indigenous creativity and quantitative ability. As Hull (1976) noted, large urban centers were disregarded by the colonialists because they did not utilize Cartesian typology. The complex fractal nature of these settlements went unappreciated. This point is crucial in any discussion of possible applications of current research. The sheer redemptive power of knowledge is at play here. Even today, people living on the African continent may still think of their indigenous past as primitive and non-rational. An understanding of the fractal characteristics of indigenous culture enables an appreciation of the complexity of the 'mundane' indigenous artifacts. This singular understanding can act as a powerful motivator for rethinking modernity.

It is widely accepted in the STS community that indigenous communities often posses tacit knowledge that may be invaluable in problem solving (Wynne 1996, Epstein 1996) but is usually deprivileged within the dominant discourse. Extending this understanding to sociopolitical and economic problems that face the continent leads us to enquire into the nature of indigenous knowledge, and its possible applications towards these challenges. We have shown, for example, that many traditional African villages were structured in a "bottom-up" process, using self-organization rather than imposed order. Could the top-down hierarchal approaches that linger on in so many post-colonial African countries – often due to the legacies of colonialism – also give way to more bottom-up self-organizing social processes? There is an obvious need for a change in the methodologies of modernity on the African continent; perhaps hybridizing indigenous mathematical representations can provide fresh thinking to a persistent problem.

The networking and scaling phenomenon that we describe may have direct applicability in political institutions. For example, in Nigeria, there is only one police organization, which is directly controlled by the federal government. Many arguments have been put forth in favor of creating more locally-controlled state police organizations, but fears of ethnic profiling and a weak center have made this a difficult proposition. Indeed this scenario is indicative of a more general political phenomenon in Nigeria; the centralization of power. This has been slowing down decision making at the local level and creating a strong reliance on central rule. While it can be pointed out that such a political structure is reminiscent of the military dictators that until very recently ruled that country, it is also true that the powerful center is a remnant of a colonial program of control. Perhaps an understanding of decentralized self-organizing systems from indigenous knowledge – not as nostalgia for an organic past, but as a computational model for the future – could assist in re-conceptualizing political power in Nigeria.

Another way the scaling phenomenon may be useful in the political field is in handling the pressing dichotomy between nation and state[2]. Perhaps a structural reorganization that recognizes the legitimacy of nationhood within the state could preserve the ethnic identities of the multiplicity of people groups in African nations (and thus relive ethnic pressures) while allowing the state to continue to exist as a fractal entity comprised of many nations of differing sizes. Frequently such visions are dismissed as mere "Balkanization" or "anarchy." We maintain that this reaction is based in part on the mistaken impression that there are only two fundamental behavioral states: centralized order and decentralized disorder. Understanding the third possibility, "decentralized order," is difficult without an exposure to its manifestations in nature, society, or technology. African fractals offer a unique opportunity to spread this understanding through indigenous culture; for example through an understanding of its religious systems (figure 4).

[2] For a well reasoned discussion of the differences between a nation and a state, see Castells (1997:30-68).

G.Sica (ed.) *What Mathematics from Africa?*

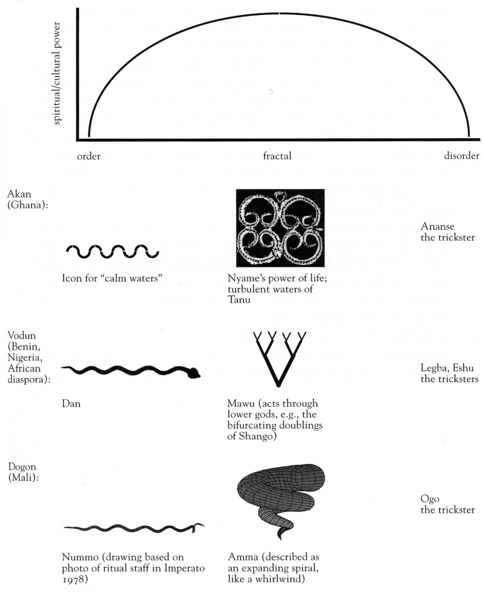


Figure 4. The spectrum from order to fractal to disorder in several African religious systems.

While analogies to religion may seem abstract or ethereal, there are many practical applications. Contemporary African architecture could utilize its fractal heritage without compromising connections to computer-based design and manufacturing (indeed possibly enhancing them). The World Wide Web has a fractal structure (Barabási 2002); could the fractal nature of African social structures be used to enhance the spread of information technology in Africa? Such "piggy backing" of social and technological networks has already been launched by one small community group, the Oke-Ogun digital bridge project in Nigeria, with some initial success (McLean 2005). There are many other possible hybrids, ranging from conceptual to concrete, social to scientific, that could result from the fusion between Africa's fractal cultural resources and its developmental efforts.

A mathematical abstraction may seem like a fragile shield against the onslaught of ecological disaster, military dictatorship and neocolonial domination that challenges African survival. But history proves that ideas can be powerful, and we are convinced that the fractal heritage of Africa holds great promise for its future.

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