1. Consider the IVP
\[(t + 1)^3y' + 4(t + 1)^2y = e^t, \quad 0 \leq t \leq 1, \quad y(0) = 0\]
(a) Find the exact solution. (Hint: consider an integrating factor for the linear ODE.)
(b) Determine \(M\) and \(L\) in the error bound for Euler’s method given by
\[
\max_{0 \leq i \leq N} |w_i - y(t_i)| \leq \frac{Mh^2}{2L} [e^{L(t_f - t_i)} - 1]
\]
(c) Compute a numerical solution using Euler’s method with \(h = 0.02\). Verify that the global error in the numerical solution satisfies the error bound.
(d) Compute a numerical solution using the second-order Taylor method described in class with \(h = 0.02\). Compute the global error and compare it with the error found for Euler’s method in part (c).

2. Consider the second-order accurate Runge-Kutta (RK) method for the ODE \(y' = f(t, y)\) given by
\[
K_1 = hf(t_i, w_i)
\]
\[
K_2 = hf(t_i + h, w_i + K_1)
\]
\[
w_{i+1} = w_i + \frac{1}{2}[K_1 + K_2]
\]
(a) Use the second-order accurate RK method to compute numerical solutions of the IVP
\[
y' = \frac{3t^2 + 4t + 2}{2(y - 1)}, \quad 0 \leq t \leq \frac{3}{2}, \quad y(0) = -1
\]
using \(h = 0.1, 0.05\) and \(0.025\). Plot the numerical solutions and the exact solution on the same graph. Be sure to label all curves. Compute the global error for each \(h\).
(b) Repeat part (a) using the standard fourth-order Runge-Kutta method (RK4). You may use the formulas for RK4 given in class, or see page 316 of the text.
(c) Estimate the rates of convergence from the computed errors in parts (a) and (b) to verify the order of accuracy of the two RK methods.

3. An integration of the ODE \(y' = f(t, y)\) from \(t_i\) to \(t_{i+1}\) leads to the equation
\[
y(t_{i+1}) = y(t_i) + \int_{t_i}^{t_{i+1}} f(t, y(t)) \, dt
\]
Following the procedure discussed in class, replace the integrand \(f\) with a quadratic polynomial fitted to the data \((t_{i-2}, f_{i-2}), (t_{i-1}, f_{i-1})\) and \((t_i, f_i)\), and then integrate the polynomial to determine the constants \((b_0, b_1, b_2)\) in the three-step method
\[
w_{i+1} = w_i + h[b_0 f(t_i, w_i) + b_1 f(t_{i-1}, w_{i-1}) + b_2 f(t_{i-2}, w_{i-2})]
\]
4. Compute an approximate solution of the IVP in Problem 2 using the Adams-Bashforth two-step method

\[ w_{i+1} = w_i + \frac{h}{2} [3f(t_i, w_i) - f(t_{i-1}, w_{i-1})] \]

\[ i = 1, 2, \ldots, N - 1 \]

As discussed in class, the two-step method requires two initial values, \( w_0 \) and \( w_1 \), to get started. Use \( w_0 = -1 \) from the initial condition in the IVP and the value for \( w_1 \) from the numerical solution of the RK method in Problem 2, part (a). Use \( h = .025 \) and compute the global error in the approximate solution. How does the global error computed here compare with that found for the RK in part (a) above? Plot the approximate solution and the exact solution on the same graph.