1. (a) Write a matlab function, myPolyfit, that computes the weights $a_i, i = 1, 2, \ldots , n$, in the Newton interpolating polynomial $\tilde{f}(x)$ of degree $n - 1$ (or less). The input to the function is the data $(x_i, y_i), i = 1, 2, \ldots , n$ and the weights should be computed based on divided differences following the algorithm discussed in class. Write a second matlab function, myPolyval, that takes the weights $\{a_i\}$, the nodes $\{x_i\}$, and a point $x$ (or vector of points) and returns the value of $\tilde{f}(x)$ using the nested iteration discussed in class.

(b) Consider the function $f(x) = \frac{2x}{1 + 20x^4}$ for $x \in [-1, 1]$

Use your matlab functions to find the weights for $\tilde{f}(x)$ fitted to the data $(x_i, y_i), i = 1, 2, \ldots , 11$, where $x_i$ are equally spaced nodes on $[-1, 1]$ and $y_i = f(x_i)$. (Hint: consider the matlab function $\text{xi=linspace(-1,1,11)}$ and the matlab examples on the course website.) Plot $f(x)$ and $\tilde{f}(x)$ on the same graph for many values of $x$ on $[-1, 1]$ (e.g. use $x=\text{linspace(-1,1,400)}$.) Comment on the behavior of the interpolating polynomial.

(c) Repeat part (b) for the Chebychev nodes $x_i = -\cos((2i - 1)\pi/22), i = 1, 2, \ldots , 11$. (Hint: consider the matlab examples on the course website.)

2. Text exercises 2 and 4 on page 156.

3. Text exercises 3 and 6 on page 165.

4. Let

$$\tilde{f}(x) = \begin{cases} P_1(x) & \text{for } x_1 \leq x < x_2 \\ P_2(x) & \text{for } x_2 \leq x \leq x_3 \end{cases}$$

where

$$P_j(x) = a_j + b_j(x - x_j) + c_j(x - x_j)^2 + d_j(x - x_j)^3, \quad j = 1 \text{ or } 2$$

(a) Write down the eight equations for the eight coefficients $(a_j, b_j, c_j, d_j), j = 1, 2$, in the natural cubic spline, $\tilde{f}(x)$, fitted to the data $(x_i, y_i), i = 1, 2, 3$.

(b) Find all coefficients (by hand) in the definition of the natural cubic spline, $\tilde{f}(x)$, that fits the data $(0, 2), (2, -4)$ and $(3, -19)$.

5. (a) Consider the data $(-1, 1), (0, -1), (1, -2)$ and $(2, -5)$. Determine a best fit line of the form $y = a_1 + a_2x$ by solving the normal equations for the coefficients $a_1$ and $a_2$.

(b) Consider the data $(-1, 1), (0, 0), (1, 0)$ and $(2, -2)$. Determine a best fit quadratic of the form $y = a_1 + a_2x + a_3x^2$ by solving the normal equations for the coefficients $a_1$, $a_2$ and $a_3$. 