
2. Consider the fixed-point iteration \( x_{n+1} = g(x_n) \), where \( g(x) = x + \ln(2 - x) \).
   
   (a) Show that \( g(x) \in [1/2, 4/3] \) whenever \( x \in [1/2, 4/3] \), and thus a fixed point of \( g(x) \) exists for \( x \in [1/2, 4/3] \).
   
   (b) Consider \( |g'(x)| \) for \( x \in [1/2, 4/3] \) to show that the fixed-point iteration converges to the unique fixed point of \( g(x) \) assuming \( x_0 \) is chosen in the interval \([1/2, 4/3]\).


4. Consider the functions
   
   \[ f_1(x) = x^3 - 2x - 5, \quad f_2(x) = e^{-x} - x, \quad f_3(x) = x \sin(x) - 1 \]
   
   (a) Write a Matlab function or script to compute a root of the function \( f(x) \) using Newton’s method. Use your code to compute the smallest positive root of each of the functions \( f_i(x) \), \( i = 1, 2, 3 \). You will need to determine a suitable starting guess, \( x_0 \), for each case, and use the stopping criterion \( |x_{n+1} - x_n| < 10^{-12} \).
   
   (b) Repeat the calculations in part (a) using the secant method. (You may use the values for \( x_1 \) obtained in part (a) along with \( x_0 \) for the two starting values needed for the secant method.) Compare the number of iterations needed to compute each roots using the secant method versus the number needed in part (a) using Newton’s method.

5. Text exercises 2(a,b) and 6, page 78.