1. Text exercises 2 and 3, page 268.

2. An approximation of \( I = \int_{a}^{b} f(x) \, dx \) given by Simpson’s rule is

\[
S_0 = S(a, b) = \frac{b - a}{6} \left[ f(a) + 4f \left( \frac{a + b}{2} \right) + f(b) \right]
\]

A second approximation of \( I \) given by the composite Simpson with two equally spaced subintervals is

\[
S_1 = S \left( a, \frac{a + b}{2} \right) + S \left( \frac{a + b}{2}, b \right)
\]

(a) Derive an estimate of the error in \( S_1 \) involving the difference \(|S_0 - S_1|\) following the approach discussed in class for the trapezoidal rule.

(b) Compute \( S_0 \) and \( S_1 \) for the integrals in text exercise 1 on page 272, and estimate the error in \( S_1 \) for each integral.

3. Find general solutions for the following ODEs. Write your solutions in the form \( y = g(t) \), for some function \( g \). (Hint: both ODEs are separable.)

\[
\begin{align*}
(a) \quad & y' = \frac{3t^2y}{1+t^3} \\
(b) \quad & y' = \frac{3t^2 - 1}{3 + 2y}
\end{align*}
\]

4. Find general solutions, \( y = (y_1(t), y_2(t))^T \), of the following systems of ODEs.

\[
\begin{align*}
(a) \quad & y' = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} y, \\
(b) \quad & y' = \begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix} y
\end{align*}
\]

5. Euler’s method for the IVP

\[
y' = f(t, y), \quad 0 < t < b, \quad y(0) = c
\]

is

\[
w_{i+1} = w_i + hf(t_i, w_i), \quad t_i = ih, \quad i = 0, 1, \ldots, N - 1, \quad w_0 = c,
\]

where \( h = b/N \) and \( N \) is the number of grid cells for the interval \([0, b] \). Write a Matlab code to find numerical solutions \( \{t_i, w_i\} \) for the following IVPs:

\[
\begin{align*}
(a) \quad & y' = \frac{3t^2y}{1+t^3}, \quad 0 < t < 2, \quad y(0) = 1, \\
(b) \quad & y' = \frac{3t^2 - 1}{3 + 2y}, \quad 0 < t < 3, \quad y(0) = 2,
\end{align*}
\]

Compute and plot solutions for the two IVPs using \( N = 50 \) and \( N = 100 \), and compute the error, \( \max_{0 \leq i \leq N} |w_i - y(t_i)| \), for each case.