1. Consider the data consisting of the four points \((-3, -7), (-1, 6), (0, 4)\) and \((2, -12)\) and the interpolating function
\[
\tilde{f}(x) = a_1\phi_1(x) + a_2\phi_2(x) + a_3\phi_3(x) + a_4\phi_4(x)
\]
Find the basis functions and weights if \(\tilde{f}(x)\) is a polynomial of degree three or less. Determine the polynomial (by hand) in two ways: (a) using Lagrange basis functions and (b) using Newton basis functions. For the Newton basis, determine the weights using a divided difference table.

2. (a) Write a matlab function, `myPolyfit`, that computes the weights \(a_i, i = 1, 2, \ldots, n\), of the Newton interpolating polynomial \(\tilde{f}(x)\) of degree \(n - 1\) (or less). The input to the function is the data \((x_i, y_i), i = 1, 2, \ldots, n\) and the weights should be computed based on divided differences following the algorithm discussed in class. Write a second matlab function, `myPolyval`, that takes the weights \(\{a_i\}\), the nodes \(\{x_i\}\), and a point \(x\) (or vector of points) and returns the value of \(\tilde{f}(x)\) using the nested iteration discussed in class.

(b) Consider the function
\[
f(x) = \frac{2}{1 + 20x^4} \quad \text{for } x \in [-1, 1]
\]
Use your matlab functions to find the weights for \(\tilde{f}(x)\) fitted to the data \((x_i, y_i), i = 1, 2, \ldots, 11\), where \(x_i\) are equally spaced nodes on \([-1, 1]\) and \(y_i = f(x_i)\). (Hint: consider the matlab function `xi=linspace(-1,1,11)` and the matlab examples on the course website.) Plot \(f(x)\) and \(\tilde{f}(x)\) on the same graph for many values of \(x\) on \([-1, 1]\) (e.g. use `x=linspace(-1,1,400)`.) Comment on the behavior of the interpolating polynomial.

(c) Repeat part (b) for the Chebychev nodes \(x_i = -\cos((2i - 1)\pi/22), i = 1, 2, \ldots, 11\). (Hint: consider the matlab examples on the course website.)

3. Text exercises 2 and 4 on page 156.

4. Text exercises 3 and 4 on page 165.