Calculus I Announcements

- The final exam is Wednesday, December 16th from 11:30-2:30 in DCC 308
- Check the course web page for the final exam study guide.
- Check the course web page for office hours.
Slide 2  Register Your iClickers

Question  Have you registered your iClicker yet?

A. Yes!
B. No, but I will register it today
C. I forgot whether I registered it, but I will go online today and re-register it.
D.
E. What’s an iClicker?
Answer to Question  Have you registered your iClicker yet?

A. Yes! is the correct answer.

B. No, but I will register it today

C. I forgot whether I registered it, but I will go online today and re-register it.

D.

E. What’s an iClicker?

• Register your iClickers (Google “register iclicker” to find the location.)

• Even if you have only used it partially or gotten all the answers wrong, it may be worth some extra credit.

• Note that the student ID is your RCS-ID (the first part of your email). (e.g. mine would be piperb)
Slide 3  **Areas and Volumes**

Explain why the following two statements are true:

1. Integrate Height times $dx$ to find Area.
2. Integrate Area times $dx$ to find Volume.
For the washer method to find volumes of solids formed by revolving around an axis parallel to the x-axis, integrate the areas of washers (times dx) with the formula

\[ \int_{a}^{b} \pi \left( (r_2(x))^2 - (r_1(x))^2 \right) \, dx \]

where

- \( r_2(x) \) is the outer radius measured perpendicularly from the axis of revolution at \( x \)
- \( r_1(x) \) is the inner radius measured perpendicularly from the axis of revolution at \( x \)

1. Visualize this method.
Slide 5   iClicker

**Question** What is the formula to find the volume of the solid of revolution formed by revolving the region bounded by \( y = x \), and \( y = x^2 \) about the line \( y = -2 \) (using washers)

A. \( \int_{0}^{1} \pi \left( (x^2 - 2)^2 - (x - 2)^2 \right) \, dx \)

B. \( \int_{0}^{1} \pi \left( (x^2 + 2)^2 - (x + 2)^2 \right) \, dx \)

C. \( \int_{0}^{1} \pi \left( (x - 2)^2 - (x^2 - 2)^2 \right) \, dx \)

D. \( \int_{0}^{1} \pi \left( (x + 2)^2 - (x^2 + 2)^2 \right) \, dx \)

E. None of the above
**Answer to Question** What is the formula to find the volume of the solid of revolution formed by revolving the region bounded by \( y = x \), and \( y = x^2 \) about the line \( y = -2 \) (using washers)

A. \( \int_{0}^{1} \pi \left( (x^2 - 2)^2 - (x - 2)^2 \right) \, dx \)

B. \( \int_{0}^{1} \pi \left( (x^2 + 2)^2 - (x + 2)^2 \right) \, dx \)

C. \( \int_{0}^{1} \pi \left( (x - 2)^2 - (x^2 - 2)^2 \right) \, dx \)

D. \( \int_{0}^{1} \pi \left( (x + 2)^2 - (x^2 + 2)^2 \right) \, dx \) **is the correct answer.**

E. None of the above
Slide 6  Solids of Revolution: Shells

• For the shell method to find volumes of solids formed by revolving around the vertical line \( x = L \), integrate the areas of cylinders (times \( dx \)) with the formula

\[
\int_a^b 2\pi (\text{Shell Radius}) \times (\text{Shell Height}) \, dx
\]

\[
= \int_a^b 2\pi (x - L) h(x) \, dx
\]

where

– \( h(x) \) is the height of the shell measured parallel to the axis of revolution at \( x \)

• Argue in more detail why

\[
2\pi (\text{Shell Radius}) \times (\text{Shell Height}) \, \Delta x
\]
is actually the volume of a thin shell of width \( \Delta x \).
Slide 7  **Shells**

Consider the region bounded by $y = x^3$, $y = 8$ and $x = 0$.

Set up, but do not evaluate integrals to:

1. Using shells, set up an integral to find the volume of the solid formed by revolving this region around the y-axis.

2. Using shells, set up an integral to find the volume of the solid formed by revolving this region around the line $x = 3$. 
Slide 8    iClicker

**Question**  What is the formula to find the volume of the solid of revolution formed by revolving the region bounded by $y = x$, and $y = x^2$ about the line $x = -2$ (using shells)?

A. $\int_{0}^{1} 2\pi (x + 2)(x^2 - x) \, dx$

B. $\int_{0}^{1} 2\pi (x + 2)(x - x^2) \, dx$

C. $\int_{0}^{1} 2\pi (x + 2)(\sqrt{x} - x) \, dx$

D. $\int_{0}^{1} 2\pi (x + 2)(x - \sqrt{x}) \, dx$

E. None of the above
Answer to Question  What is the formula to find the volume of the solid of revolution formed by revolving the region bounded by $y = x$, and $y = x^2$ about the line $x = -2$ (using shells)?

A. $\int_0^1 2\pi (x + 2)(x^2 - x) \, dx$

B. $\int_0^1 2\pi (x + 2)(x - x^2) \, dx$  is the correct answer.

C. $\int_0^1 2\pi (x + 2)(\sqrt{x} - x) \, dx$

D. $\int_0^1 2\pi (x + 2)(x - \sqrt{x}) \, dx$

E. None of the above
Final Exam, Possible Essay Question 1

State the definition of derivative. Illustrate the definition with a picture and explain how your picture shows that the derivative gives the instantaneous slope of the graph.
1. Draw a well labelled picture to illustrate the right sum
\[ \sum_{k=1}^{n} f(x_k) \triangle x \] for a general continuous, positive
function \( y = f(x) \) using equally spaced subintervals on
an interval \([a, b]\).

2. Referring to your picture, explain how the above sum
approximates the area of the region under the graph
of \( y = f(x) \), above the \( x \) axis and between \( x = a \) and
\( x = b \).

3. Explain how to make this approximation give the
definite integral and the exact area under the curve.
1. State the first part of the Fundamental Theorem of Calculus. (Integration of the Derivative.)

2. Give a major application for this part of the theorem and show an example.

3. Using the basic operations of addition, subtraction, multiplication and division involved in the definitions of integration and differentiation, explain why differentiation undoes integration.
1. State the second part of the Fundamental Theorem of Calculus (Differentiation of the Integral.)

2. Give a major application for this part of the theorem and show an example.

3. Using the basic operations of addition, subtraction, multiplication and division involved in the definitions of integration and differentiation, explain why integration undoes differentiation.