Calculus I Announcements

- Exam 4 is Monday, December 7th
- See the study guide on the course web page.
- See the next page for more information.
In working the exercises on the study guide for the exam, certain formulas arise. There will not be a formula sheet on the exam, so you will be expected to know any formulas that you may need. In general, you should know any formula listed in the summary of any section on the exam. The list below highlights a few of the formulas that students frequently forget and a few that you do not have to memorize. It is not comprehensive of all the formulas you may need to work the exam.

1. \[ \sum_{j=1}^{N} c = N \cdot c \]

2. \[ \sum_{j=1}^{N} j = \frac{N(N + 1)}{2} \]

3. \[ \sum_{j=1}^{N} j^2 = \frac{N(N + 1)(2N + 1)}{6} \]

4. You do not need to memorize the formula for the sum of cubes.

5. \[ \int b^x = \frac{b^x}{\ln(b)} + C \]

6. \[ \int \frac{dx}{\sqrt{1 - x^2}} = \sin^{-1}(x) + C \]

7. \[ \int \frac{dx}{x^2 + 1} = \tan^{-1}(x) + C \]

8. You do not need to memorize \[ \int \frac{dx}{|x|\sqrt{x^2 - 1}} = \sec^{-1}(x) + C \]
Slide 3   Finding Areas

Content on this slide is for Exam 4

- Explain the statement:
  “Integrate height times $dx$ to find area.”
Content on this slide is for Exam 4

For the problems below, set up, but do not evaluate integrals, with respect to $x$ to find the indicated areas.

1. **Four Boundaries** Find the area of the region bounded by $y = x^2$, and $y = x$, and $x = 2$ and $x = 3$.

2. **Two Boundaries** Find the area of the region bounded by $y = x^2$ and $y = 3 - 2x$.

3. **Boundaries that cross over** Find the area of the region between $y = \sin(x)$ and $y = \cos(x)$ for $x$ between $0$ and $\pi$.

**Method**

1. Sketch the Region.

2. Determine the top curve and bottom curve.

3. Sketch a characteristic rectangle and determine its height to find the integrand.

4. Determine the limits of integration.

5. Set up the integral.

6. Integrate.
Question Set up, but do not evaluate, an integral for the region that lies between the curves $y = 1$ and $y = (\sin(x))^2$ and between $x = -\pi/2$ and $x = \pi/2$

A. $\int_{-\pi/2}^{\pi/2} \left( 1 - (\sin(x))^2 \right) \, dx$

B. $\int_{-\pi/2}^{\pi/2} \left( (\sin(x))^2 - 1 \right) \, dx$

C. $\int_{0}^{1} \left( (\sin(x))^2 - \pi/2 \right) \, dx$

D. $\int_{0}^{1} \left( \pi/2 - (\sin(x))^2 \right) \, dx$

E. None of the above
**Answer to Question** Set up, but do not evaluate, an integral for the region that lies between the curves $y = 1$ and $y = (\sin(x))^2$ and between $x = -\pi/2$ and $x = \pi/2$

A. $\int_{-\pi/2}^{\pi/2} \left( 1 - (\sin(x))^2 \right) dx \quad \text{is the correct answer.}$

B. $\int_{-\pi/2}^{\pi/2} \left( (\sin(x))^2 - 1 \right) dx$

C. $\int_{0}^{1} \left( (\sin(x))^2 - \pi/2 \right) dx$

D. $\int_{0}^{1} \left( \pi/2 - (\sin(x))^2 \right) dx$

E. None of the above
Finding Areas

Content on this slide is for Exam 4

For some problems, it is necessary (or much easier) to integrate with respect to $y$ (instead of $x$). Try this on the problems below.

1. **Two Boundaries** Find the area of the region bounded by $x = y^2$, and $x = y^3$.

2. **Three Boundaries** Find the area of the region bounded by $y = \sqrt{x}$, and $x = -\sin(y)$ and $y = 1$.

Method

1. Sketch the Region.

2. Determine the right curve and left curve.

3. Sketch a characteristic (horizontal) rectangle and use it to find the integrand.

4. Determine the limits of integration.

5. Set up the integral.

6. Integrate.
Slide 7  Finding Areas and Volumes

Content on this slide is for the Final Exam, not Exam 4

Areas and volumes of curved regions can be found by using the definite integral. The central ideas are:

1. Integrate Height times dx to find Area.
2. Integrate Area times dx to find Volume.
Slide 8  **Volumes of Solids of Revolution**

Content on this slide is for the Final Exam, not Exam 4

**Disk Method** We use this idea to find volumes of solids of revolutions by integrating areas of disks as in the following example.

1. Sketch a picture of the region under the curve $y = x^2$, above the $x$-axis and between $x = 0$ and $x = 1$.

2. Visualize and/or sketch a picture of the solid formed by revolving this region about the $x$-axis.

3. Visualize and/or sketch a cross sectional disk formed from the revolution.

4. Write down the radius of the disk as a function of $x$

5. Write down the Area of the disk as a function of $x$

6. Integrate the area times $dx$ to get the volume.
Content on this slide is for the Final Exam, not Exam 4

Question  Find the volume of the solid of revolution formed by revolving the region bounded by $y = \sqrt{x}$, the x-axis, $x = 1$ and $x = 3$ about the x-axis.

A. $\pi/3$
B. $4\pi/3$
C. $\pi/2$
D. $3\pi/2$
E. None of the above
**Answer to Question**  Find the volume of the solid of revolution formed by revolving the region bounded by $y = \sqrt{x}$, the x-axis, $x = 1$ and $x = 3$ about the x-axis.

A. $\pi/3$

B. $4\pi/3$

C. $\pi/2$

D. $3\pi/2$

**E. None of the above** is the correct answer.

The answer is $4\pi$
Washer Method: Horizontal Axes

Content on this slide is for the Final Exam, not Exam 4

To find the volume of a region revolved about an axes parallel to the $x$ axis, try the following steps.

1. Sketch the region and find the limits of integration.
2. Try to visualize the revolution and the solid.
3. Find the inner radius $r_1(x)$ as a function of $x$ by finding the vertical distance from the inner function to the axis of revolution.
4. Find the outer radius $r_2(x)$ as a function of $x$ by finding the vertical distance from the outer function to the axis of revolution.
5. Use the formula
   \[ V = \int_{a}^{b} \pi ((r_2(x))^2 - (r_1(x))^2) \, dx \]

Example
Set up (but do not integrate) an integral for the volume of the solid of revolution formed by revolving the region bounded by $y = x^2$ and $y = 2$ around the line $y = 5$. 
Question  What is the formula to find the volume of the solid of revolution formed by revolving the region bounded by $y = \sqrt{x}$, and $y = x^2$ about the x-axis.

A. $\int_0^1 \pi (\sqrt{x} - x^2)^2 \, dx$

B. $\int_0^1 \pi \left( (\sqrt{x})^2 - (x^2)^2 \right) \, dx$

C. $\int_0^1 \pi \left( (x^2)^2 - (\sqrt{x})^2 \right) \, dx$

D. $\int_0^1 \pi (x^2 - \sqrt{x})^2 \, dx$

E. None of the above
**Answer to Question**  What is the formula to find the volume of the solid of revolution formed by revolving the region bounded by $y = \sqrt{x}$, and $y = x^2$ about the x-axis.

A. $\int_0^1 \pi (\sqrt{x} - x^2)^2 \, dx$

B. $\int_0^1 \pi \left( (\sqrt{x})^2 - (x^2)^2 \right) \, dx$  is the correct answer.

C. $\int_0^1 \pi \left( (x^2)^2 - (\sqrt{x})^2 \right) \, dx$

D. $\int_0^1 \pi (x^2 - \sqrt{x})^2 \, dx$

E. None of the above
Content on this slide is for the Final Exam, not Exam 4

Question What is the formula to find the volume of the solid of revolution formed by revolving the region bounded by $y = x$, and $y = x^2$ about the line $y = -2$ (using washers)

A. $\int_{0}^{1} \pi \left( (x^2 - 2)^2 - (x - 2)^2 \right) \, dx$

B. $\int_{0}^{1} \pi \left( (x^2 + 2)^2 - (x + 2)^2 \right) \, dx$

C. $\int_{0}^{1} \pi \left( (x - 2)^2 - (x^2 - 2)^2 \right) \, dx$

D. $\int_{0}^{1} \pi \left( (x + 2)^2 - (x^2 + 2)^2 \right) \, dx$

E. None of the above
Answer to Question  What is the formula to find the volume of the solid of revolution formed by revolving the region bounded by $y = x$, and $y = x^2$ about the line $y = -2$ (using washers)

A. $\int_0^1 \pi ((x^2 - 2)^2 - (x - 2)^2) \, dx$

B. $\int_0^1 \pi ((x^2 + 2)^2 - (x + 2)^2) \, dx$

C. $\int_0^1 \pi ((x - 2)^2 - (x^2 - 2)^2) \, dx$

D. $\int_0^1 \pi ((x + 2)^2 - (x^2 + 2)^2) \, dx$  is the correct answer.

E. None of the above