Calculus I Announcements

- Read sections 5.1-5.4, and 5.7, 5.8 and do the homework from these.
1. The **indefinite integral** is written $\int f(x) \, dx$ and means to take the anti-derivative.

2. The **definite integral** is written $\int_{a}^{b} f(x) \, dx$ and is defined by taking the limit Riemann Sums as the partition size gets small and the number of rectangles goes to infinity. The function $f$ is called the integrand and $x$ is a “dummy variable”.

$$\int_{a}^{b} f(x) \, dx = \lim_{||P|| \to 0} \sum_{i=1}^{n} f(c_i) \Delta x_i$$

3. The Fundamental Theorem of Calculus provides a connection between these two concepts for continuous integrands $f$:

If $F(x) = \int f(x) \, dx$ then $\int_{a}^{b} f(x) \, dx = F(b) - F(a)$
FTC: Integration of the Derivative If $f$ is continuous on $[a, b]$ and $F$ is an anti-derivative of $f$ then

$$\int_a^b f(x) \, dx = F(b) - F(a)$$

1. Use this to find the area under $y = \sin(x)$ over the interval $[0, \pi]$
2. Explain why this is called integration of the derivative.
3. Show that it does not matter which anti-derivative $F$ we choose.
Question: Find \( \int_{1}^{3} \frac{x^2 + 1}{x} \, dx \)

A. 10
B. ln(3)
C. ln(3) + 4
D. \( \frac{3}{2} \)
E. None of the above
Answer to Question  Find \( \int_{1}^{3} \frac{x^2 + 1}{x} \, dx \)

A. 10
B. \( \ln(3) \)
C. \( \ln(3) + 4 \) is the correct answer.
D. \( \frac{3}{2} \)
E. None of the above
Discuss the following properties of integrals and illustrate as appropriate:

1. \( \int_a^b 0 \, dx = 0 \)

2. \( \int_a^b (f(x) + g(x)) \, dx = \int_a^b f(x) \, dx + \int_a^b g(x) \, dx \)

3. \( \int_a^b \alpha f(x) \, dx = \alpha \int_a^b f(x) \, dx \) for any constant \( \alpha \)

4. \( \int_a^b f(x) \, dx = \int_a^c f(x) \, dx + \int_c^b f(x) \, dx \)

5. \( \int_a^b f(x) \, dx = -\int_b^a f(x) \, dx \)

6. If \( m \leq f(x) \leq M \) then
   \[ m(b - a) \leq \int_a^b f(x) \, dx \leq M(b - a) \]

7. If \( f(x) \leq g(x) \) for all \( x \) then \( \int_a^b f(x) \, dx \leq \int_a^b g(x) \, dx \)

8. \( \int_a^b f(x) \, dx \leq \int_a^b |f(x)| \, dx \)
1. Show that \( \int_{0}^{1} \sin(\pi x^3) \, dx \leq 1 \)

2. Show that \( \int_{0}^{2} e^{x^2} \, dx \geq 1 + e \)
Question Suppose $\int_{0}^{2} f(x) \, dx = 3$ and $\int_{0}^{4} f(x) \, dx = 5$, what is $\int_{2}^{4} f(x) \, dx$?

A. $-8$
B. 8
C. 2
D. $-2$
E. None of the above
Answer to Question  Suppose $\int_{0}^{2} f(x) \, dx = 3$ and $\int_{0}^{4} f(x) \, dx = 5$, what is $\int_{2}^{4} f(x) \, dx$?

A. $-8$

B. $8$

C. $2$ is the correct answer.

D. $-2$

E. None of the above
Example Consider

\[ \int \cos(x^2)2x \, dx \]

Let

\[ u = x^2 \quad du = 2x \, dx \]

\[ \int \cos(x^2)2x \, dx = \int \cos(u) \, du = \sin(u) + C = \sin(x^2) + C \]

The general form of this example is with \( u = g(x) \) and \( du = g'(x) \, dx \)

\[ \int F'(g(x))g'(x) \, dx = \int F'(u) \, du = F(u) + C = F(g(x)) + C \]

1. Find \( \int \frac{t^3}{1 + t^4} \, dt \)
2. u-substitution undoes what differentiation rule?
Question  Find $\int \frac{e^x}{e^x + 2} \, dx$ using $u$ substitution.

A. $e^x + 2 + C$
B. $x + 2 + C$
C. $\ln(e^x + 2) + C$
D. $\ln(e^x) + \ln(2) + C$
E. None of the above
**Answer to Question**  
Find $\int \frac{e^x}{e^x + 2} \, dx$ using $u$ substitution.

A. $e^x + 2 + C$
B. $x + 2 + C$
C. $\ln(e^x + 2) + C$ is the correct answer.
D. $\ln(e^x) + \ln(2) + C$
E. None of the above
Example Consider

\[
\int_0^{\sqrt{\pi/2}} \cos(x^2)2x\,dx
\]

- \( u = x^2 \)
- \( du = 2x\,dx \)
- Change the limits of integration
  - Lower: when \( x = 0 \) then \( u = 0^2 = 0 \)
  - Upper: when \( x = \sqrt{\pi/2} \) then \( u = (\sqrt{\pi/2})^2 = \pi/2 \)

so

\[
\int_0^{\sqrt{\pi/2}} \cos(x^2)2x\,dx = \int_0^{\pi/2} \cos(u)\,du = \sin(\pi/2) - \sin(0) = 1 - 0 = 1
\]

The general form of this example is with \( u = g(x) \) and \( du = g'(x)\,dx \)

\[
\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du
\]
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Question \( \int_0^{\pi/2} (\cos(x))^4 \sin(x) \, dx \) equals

A. \( \frac{1}{5} \)

B. \( \frac{-1}{5} \)

C. \( \frac{\pi}{10} \)

D. \( \frac{-\pi}{10} \)

E. None of the above
Answer to Question \[ \int_0^{\pi/2} (\cos(x))^4 \sin(x) \, dx \] equals

A. \( \frac{1}{5} \) is the correct answer.

B. \( -\frac{1}{5} \)

C. \( \frac{\pi}{10} \)

D. \( -\frac{\pi}{10} \)

E. None of the above
**Question** If \( A = \int_{0}^{1} (1 + 2x^2)^{45} 4x \, dx \), which of the following is false?

A. \( A \) can be evaluated using the \( u \) substitution \( u = 1 + 2x^2 \)

B. \( A = \int_{0}^{1} u^{45} \, du \)

C. \( A \) is the area under the curve \( y = (1 + 2x^2)^{45} 4x \) between \( x = 0 \) and \( x = 1 \)

D. \( A \) is the area under the curve \( y = u^{45} \) between \( x = 1 \) and \( x = 3 \)

E. \( A = \frac{3^{46} - 1}{46} \)
Answer to Question  If $A = \int_{0}^{1} (1 + 2x^2)^{45} 4x \, dx$, which of the following is false?

A. $A$ can be evaluated using the $u$ substitution $u = 1 + 2x^2$

B. $A = \int_{0}^{1} u^{45} \, du$ is the correct answer.

C. $A$ is the area under the curve $y = (1 + 2x^2)^{45} 4x$ between $x = 0$ and $x = 1$

D. $A$ is the area under the curve $y = u^{45}$ between $x = 1$ and $x = 3$

E. $A = \frac{3^{46} - 1}{46}$
Question \( \int_{0}^{1} \frac{t}{1 + t^4} \, dt \) equals

A. \( \pi/8 \)
B. \( \ln(2) \)
C. \( \arctan(2) - \arctan(1) \)
D. \( \pi/4 \)
E. None of the above
Answer to Question \( \int_0^1 \frac{t}{1 + t^4} \, dt \) equals

A. \( \pi/8 \) is the correct answer.

B. \( \ln(2) \)

C. \( \arctan(2) - \arctan(1) \)

D. \( \pi/4 \)

E. None of the above

We make the substitution \( u = t^2 \) and use the formula

\[ \int \frac{1}{1 + x^2} \, dx = \arctan(x) + C \]