``I think you're begging the question,'' said Haydock, ``and I can see looming Ahead one of those terrible exercises in probability where six men have white hats and six men have black hats and you have to work it out by mathematics how likely it is that the hats will get mixed up and in what proportion. If you start thinking about things like that, you would go round the bend. Let me assure you of that!''

Christie, Agatha  The Mirror Crack'd

Lecture 5: Independence

Probability Theory and Applications
Fall 2008
September 11

August 28, 2008
Outline

- Independence
- Review of Bayes Rule
- In-class Exercise
Breaking Down Events

• Calculating probabilities can be easier, if we break down the event into pieces whose probabilities.

Event A: student is female
Event B: student is male
A and B are mutually exclusive events since

\[ A \cap B = \emptyset \]
Mutually Exclusive Events

If A and B are mutually exclusive then

\[ P(A \cup B) = P(A) + P(B) \]
Independence

If A and B are independent, then knowing A tells you nothing about B

Def: A and B are independent iff

\[ P(AB) = P(A) \times P(B) \]

Independent or not:
Rolling two dice,
A=rolling 1 on first dice, B=rolling 1 on second dice?

Eating jelly beans from a jar with 30 strawberry and 30 grape jellybeans,
A= First jelly bean eaten is strawberry, B=Second jelly eaten is strawberry?
Example

• Consider a family with two children
• A = first kid girl
• B = second kid girl
• C = family has 1 boy and 1 girl

Are A and B independent?
Are A and C independent?
Are B and C independent?
Details

- \( P(A) = 0.5 \quad P(B) = 0.5 \)
- \( P(AB) = P(\text{Girl}, \text{Boy}) = 0.5 \times 0.5 = 0.25 = P(A)P(B) \)
- \( A \) and \( B \) are independent

- \( P(AC) = P(\text{Girl}, \text{Boy}) = P(A)P(C) = 0.5 \times 0.5 \)
- \( A \) and \( C \) are independent
- You check \( B \) and \( C \).
Complete Independence

Are A, B, C completely independent?

\[ P(ABC) = P(\text{Girl, Boy}) = 0.25 \]
\[ P(A) \times P(B) \times P(C) = 0.5 \times 0.5 \times 0.5 \neq 0.25 \]

A, B, C are pairwise independent but not completely independent.
Are A and B independent?
Are C and D independent?
Handy Facts

If A and B are independent then

1) $A$ and $\overline{B}$ are independent
3) $\overline{A}$ and $B$ are independent
2) $\overline{A}$ and $\overline{B}$ are independent

Proof of 1) 

\[
P(AB) = P(A) - P(AB) = P(A) - P(A)P(B) = P(A)[1 - P(B)] = P(A)P(\overline{B})\]

\[\square\]
Example

Consider a twin engine plane. Assume engines are independent. The probability of engine failure of a single engine is $10^{-4}$. What is probability that both engines fail?

Is this a good assumption?
General Bayes Rule

- Let $A_1, A_2, \ldots, A_n$ be disjoint events that form a partition of the sample space

\[
P(A_i \mid B) = \frac{P(B \mid A_i)P(A_i)}{\sum_j P(B \mid A_j)P(A_j)}
\]