An Adversarial Hierarchical Hidden Markov Model for Human Pose Modeling and Generation

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Introduction

• Generative dynamic model: capture the data distribution change over time
  • Major categories:
    • Directed Probabilistic Graphical Models: HMM, DBN
    • Undirected Probabilistic Graphical Models: TRF, TRBM
    • Neural Networks: RNN, extension of VAE, GAN
  • Major tasks:
    • Representation learning
    • Data synthesis
    • Data imputation
Methods: Base Model

- Hidden Markov Model
  - Random variables: $X_t, Z_t$
  - Joint distribution:

$$P(X, Z) = P(X_0) \prod_{t=1}^{T} P(X_t|Z_t)P(Z_t|Z_{t-1})$$

Initial  Emission  Transition

```
Z_0 -> Z_1 -> Z_2 -> Z_3 -> ... -> Z_T
```

```
X_1 -> X_2 -> X_3 -> ... -> X_T
```

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Methods: Base Model

• Hidden Markov Model
  • Random variables: $X_t, Z_t$
  • Joint distribution:
    $$P(X, Z|\theta) = P(X_0|\pi) \prod_{t=1}^{T} P(X_t|Z_t, \psi)P(Z_t|Z_{t-1}, A)$$
  • Parameters: $\theta = \{\pi, A, \psi\}$
Methods: Model

• Bayesian Hierarchical Hidden Markov Model
  • Random variables: $X_t, Z_t$
  • Joint distribution: $P(X, Z|\alpha) = \int P(X, Z|\theta)P(\theta|\alpha)d\theta$
  • Parameters as random variables: $\theta = \{A, \psi\}$
  • Hyperparameters: $\alpha = \{\pi, \eta, \lambda\}$
Methods: Model Learning

• Goal: estimate the values of hyperparameters $\alpha$
  • Conventional: Maximum Likelihood (ML)
    \[
    \alpha^* = \arg \max_{\alpha} \log P(\mathbf{X}|\alpha)
    \]
    \[
    = \arg \max_{\alpha} \log \int_\theta \sum_{\mathbf{Z}} P(\mathbf{X}, \mathbf{Z}|\theta) P(\theta|\alpha) d\theta
    \]

• Issue:
  • Intractability: integration over $\theta$ introduces additional dependency among $\mathbf{Z}$. Exact evaluation intractable.
  • Diffusion: tends to fit a diffused distribution in order to cover all the observed data.
Methods: Model Learning

• Proposed: Adversarial Learning (AL)
  • A two-player ‘game’
  • Generator: generate data looks as realistic as possible
    \[ \mathbf{X} \sim P_G(\mathbf{X}) \]
  • Discriminator: differentiate synthetic data from the real
    \[ P_D(y|\mathbf{X}), \ y = \begin{cases} 
      1, & \text{if } \mathbf{X} \text{ is real} \\
      -1, & \text{otherwise}
    \end{cases} \]

• The overall objective

\[
\min_{\alpha} \max_{\phi} \mathbb{E}_{\mathbf{X} \sim P_{data}(\mathbf{x})} [\log D(\mathbf{X}|\phi)] + \mathbb{E}_{\mathbf{X} \sim P_G(\mathbf{x}|\alpha)} [\log(1 - D(\mathbf{X}|\phi))] \\
D(\mathbf{X}|\phi) \triangleq P_D(y = 1|\mathbf{X}, \phi)
\]

**Theorem** [Goodfellow et al. 2014]: Given the optimal discriminator, the optimal generator minimizes the Jensen-Shannon Divergence (JSD) between data distribution and model distribution.
Methods: Optimization

• Optimize Discriminator while holding Generator fixed
• Discriminator: a pair of HMMs: $P(X|\phi^+)$ and $P(X|\phi^-)$

$$\max_{\phi} L_D(\phi) \triangleq \mathbb{E}_{x \sim P_{data}(x)}[\log D(X|\phi)] + \mathbb{E}_{x \sim P_{G}(x|\alpha)}[\log(1 - D(X|\phi))]$$

$$D(X|\phi) = \frac{P(X|\phi^+)}{P(X|\phi^+) + P(X|\phi^-)}$$
Methods: Optimization

• Optimize Generator while holding Discriminator fixed
  • Generator: HHMM: $P(\mathbf{X}|\alpha)$

$$\max_{\alpha} L_G(\alpha) \triangleq \mathbb{E}_{\mathbf{X} \sim P_G(\mathbf{X}|\alpha)}[\log D(\mathbf{X}|\phi)]$$

Diagram:

- $P(\mathbf{X}|\alpha)$ (Generator)
- Synthetic data
- $P(\mathbf{X}|\phi^+)$ (Discriminator)
- $P(\mathbf{X}|\phi^-)$
- $P(y|\mathbf{X})$
Methods: Optimization

• Update $\alpha$ and $\phi$ using SGD + RMSProp

• Compute gradient $\frac{\partial L_G}{\partial \alpha}$ and $\frac{\partial L_D}{\partial \phi}$ with
  - MC estimate of expectation
  - $\nabla_x f(x) = f(x) \nabla_x \log f(x)$

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**Algorithm 1** Adversarial learning of HHMM

**Require:** \{X\}: real dataset. $Q$: number of hidden states. $M$: number of samples. $N$: number of parameter sets. $k$: update step for $\phi$. $l$: update step for $\alpha$.

**Ensure:** Generator $\alpha$. Discriminator $\phi$.

1: Initialization of $\alpha$, $\phi$
2: repeat
3: \hspace{1em} for $k$ steps do
4: \hspace{2em} Draw $M$ samples from both $P_G$ and real dataset.
5: \hspace{2em} Update discriminator $\phi$ using RMSProp with gradient defined by Eq. (8) and Eq. (9).
6: \hspace{1em} end for
7: \hspace{1em} for $l$ steps do
8: \hspace{2em} Draw $N$ samples of $\theta$. For each $\theta$, draw $M$ samples.
9: \hspace{2em} Update generator $\alpha$ using RMSProp with gradient defined by Eq. (4) and Eq. (5).
10: \hspace{1em} end for
11: until convergence or reach maximum iteration number
12: return $\alpha$
Methods: Inference

• Data synthesis: generate novel data
  • Ancestral sampling of hidden state
    \[ \theta \sim P(\theta | \alpha^*) \]
    \[ Z_0 \sim P(Z_0 | \theta) \]
    \[ Z_t \sim P(Z_t | Z_{t-1}, \theta) \]
  • Compute the most likely observations [Brand 1999]
    \[ X^* = \arg\max_X \log P(\tilde{X}|Z, \theta) = \sum_t \log \mathcal{N}(\tilde{X}_t | \mu_{Z_t}, \Sigma_{Z_t}) \]
    where \[ \tilde{X}_t = [X_t, X_t - X_{t-1}] \]
Experiments: Data

• Dataset:
  • CMU Motion Capture (subset): walking, running, boxing. 101 sequences on average per action.
  • Berkeley MHAD (subset): jumping, jack, boxing. 60 sequences per action.

• Representation: joint angles (CMU: 53, Berkeley: 60)
Experiments: Results

- Data synthesis: qualitative results

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<th>Jumping</th>
<th>Boxing</th>
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Experiments: Results

• Data Synthesis: Quantitative results
  • Metric: Average maximum Structural Similarity Index (SSIM) [Wang et al. 2004]
  • Meaning: measure overall diversity of the synthetic data. The lower the better
Conclusion

• Proposed a Bayesian hierarchical extension to HMM to allow large modeling capacity
• Developed an adversarial learning based method for estimating the model hyperparameters
• Demonstrated the method effectiveness on motion capture data synthesis and reconstruction.

• Future extension
  • Unsupervised learning of the model
  • Full Bayesian inference
  • Incorporate long-term dependencies
Thank You

Q&A

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