Application of microwave detection of the Shubnikov–de Haas effect in two-dimensional systems

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Microwave detection of the Shubnikov–de Haas (SdH) effect as a contact-free characterization technique for different types of two-dimensional semiconductor structures is explored in the low magnetic field region. The detection technique and the data analysis are described. The character and relevance of the single-particle relaxation time that can be detected by this technique are distinguished from the usual transport scattering time. The measured values of the carrier concentration and single-particle relaxation time agree with electrical measurements, while the problem of making contacts on the structure is avoided. Uncertainties in the analysis for the single-particle relaxation time are discussed. Cyclotron resonance, optically detected cyclotron resonance, and magneto-photoluminescence are applied as other contact-free techniques on the same samples. The results and suitability of these techniques are compared with the microwave detection of the SdH effect.

I. INTRODUCTION

During recent years much efforts has been expended to optimize two-dimensional electron gas structures (2DEG) and quantum wells for the purpose of fundamental research as well as for device applications. The developments in advanced epitaxy, and the success of new sample structures, are often evaluated by the increase of the mobility of the carriers, $\mu = e\tau / m^*$. In order to judge whether the mobility is always the relevant property to describe the quality of a sample, one has to understand the physical character of $\tau$. The transport scattering time $\tau$ is identical with the momentum relaxation time in the Boltzmann approximation. It is connected to the differential scattering cross section $\sigma_{sc}(\theta)$ by

$$\frac{1}{\tau} = \int_{-\pi}^{\pi} \sigma_{sc}(\theta) (1 - \cos \theta) d\theta.$$  \hspace{1cm} (1)

The weighting factor $(1 - \cos \theta)$ takes into consideration the fact that the momentum and velocity of a particle in the original direction are decreased by a scattering event through an angle $\theta$, only by a factor of $\cos \theta$. Consequently, by measuring $\tau$, one counts predominantly large-angle scattering events and assumes small-angle scattering events to be less important. However, for many interesting applications, like experiments in the quasiballistic and ballistic regime, or of the fractional quantum Hall effect, it is not clear whether the mobility always describes the suitability of a sample.\(^2\) A property that counts the total scattering probability is the single-particle relaxation time $\tau_s$ defined by

$$\frac{1}{\tau_s} = \int_{-\pi}^{\pi} \sigma_{sc}(\theta) d\theta.$$  \hspace{1cm} (2)

While $\tau_i$ is a measure of momentum relaxation and thus of the mobility, $\tau_s$ can be identified as the lifetime of an electronic eigenstate, e.g., a Landau state in a magnetic field.\(^1\)\(^3\)

The distinction between the two scattering times is important, since $\tau_i$, in the presence of small-angle scatterers, is significantly larger\(^1\) than $\tau_s$. Experiments give for the ratio $\tau_i / \tau_s$ typical values of 6 to 14 in GaAs/AlGaAs heterojunctions, but can exceed 100 in some high mobility samples.\(^2\) In GaAs/AlGaAs quantum wells, values of 2 to 20 have been obtained by Bockelmann et al.\(^5\) Furthermore, the two relaxation times can be limited by different scattering mechanisms,\(^5\) giving the risk that optimizing the mobility does not affect the total scattering probability.

Mobilities are usually determined by Hall measurements in the classical regime while the single-particle relaxation time can be deduced from Shubnikov–de Haas (SdH) measurements. Commonly, one detects SdH oscillations electrically as oscillations of the resistivity. To perform such experiments, ohmic contacts to the 2DEG or the quantum well are required. The contacting procedure is time consuming, often technologically difficult to perform, and is destructive of the material. A detection technique that does not need contacts is therefore desirable.

Microwave detection of SdH oscillations\(^6\) provides this advantage. In Sec. II we will describe this experimental technique in detail, describe the data analysis for single-particle relaxation times, carrier concentrations and effective masses, and report experimental results obtained from different two-dimensional systems. For comparison, other contact-free characterization techniques, such as cyclotron resonance, optically detected cyclotron resonance (ODCR), and magnetophotoluminescence have been applied to the same sample. A short description of these techni-
A magnetic field $B$, applied perpendicular to the surface, forces the electrons to move in quantized, circular orbits in the two-dimensional plane, causing a splitting of each subband into a series of Landau levels.$^7$

The density of states $g(E)$ of a two-dimensional electron system in a magnetic field is a superposition of broadened Landau levels with equal degeneration (Fig. 1). It results in a constant background density of states $g_0$, with an oscillatory part, $A g(E)$ dependent on the magnetic field. Ishihara and Smrcka$^{11}$ derive for $A g(E)$

$$A g(E) = 2g_0 \sum_{s=1}^{\infty} \exp \left(-\frac{\pi}{\omega_c \tau_s} \right) \cos \left(2\pi \frac{E}{\hbar \omega_c} s - n \pi \right).$$

In the derivation, the time $\tau$ in the exponential factor is directly connected to a level broadening$^{13}$ $\Gamma = \hbar/\tau$ and is thus easily identified as the single-particle relaxation time $\tau_s$. The exponential increase of the oscillation and its dependence on the lifetime of the Landau state was first derived by Dingle,$^{14}$ and the factor is called the Dingle factor. For intermediate magnetic fields ($\omega_c \tau_s \approx 1$) we can neglect the higher terms ($s > 1$) in Eq. (4) and write

$$\frac{A g(E)}{g_0} = 2 \exp \left(-\frac{\pi}{\omega_c \tau_s} \right) \cos \left(2\pi \frac{E}{\hbar \omega_c} \right),$$

where the phase $\pi$ has been ignored. The DOS is now approximated as a constant background with a sinusoidal, oscillating part. Noting that $\omega_c = eB/m^*e$, we recognize the characteristic periodic behavior in $1/B$.

Coleridge et al.$^{12}$ stress that $\tau_s$ appears in the expressions for the conductivity only through the DOS. They show that in the derivation of Ishihara and Smrcka,$^{11}$ all other scattering times that appear in the result are the transport scattering time $\tau_t$. The result is then

$$\sigma_{xx}(B) = \sigma_0 \frac{\pi}{\omega_c \tau_s} \left[ 1 + \frac{(\omega_c \tau_s)^2}{1 + (\omega_c \tau_s)^2} D(X) \right] \frac{\exp \left(-\frac{\pi}{\omega_c \tau_s} \right) \cos \left(2\pi \frac{E_F}{\hbar \omega_c} \right)}{1 + 4 \left(\frac{\omega_c \tau_s}{1 + (\omega_c \tau_s)^2} \right)^2 D(X)},$$

where $D(X)$ is a function of the magnetic field. Isihara and Smrcka$^{11}$ later came to equivalent results by different methods. Both theories take into account only short-range scatterers with a scattering cross section which is independent of angle. The two scattering times $\tau_s$ and $\tau_t$ then become identical, and consequently the theoretical expressions contain only one phenomenological scattering time $\tau$. In real samples however, long-range scatterers like remote impurities, interface roughnesses, and alloy fluctuations are present, causing predominantly small-angle scattering events.

For this reason it is necessary to discuss the theoretical results and identify the single-particle scattering time $\tau_s$, which is to be determined. The interpretation of the scattering times in Ando's and Ishihara's expressions have been the subject of several investigations.$^{3,5,12}$ We follow here the conclusions of Coleridge et al., who confirmed their plausibility arguments by convincing experimental results.$^{2,12}$

As already mentioned, the density of states $g(E)$ of a two-dimensional electron system in a magnetic field is a superposition of broadened Landau levels with equal degeneration (Fig. 1). It results in a constant background DOS, $g_0$, with an oscillatory part, $A g(E)$ dependent on the magnetic field. Ishihara and Smrcka$^{11}$ derive for $A g(E)$

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\[ \sigma_{xy}(B) = -\frac{\pi \omega \xi^2}{1 + (\omega \xi)^2} \left[ 1 - \frac{6(\omega \xi)^2 + 1}{(\omega \xi)^2[1 + (\omega \xi)^2]} \right] \times D(X) \exp \left( -\frac{\pi}{\omega \xi} \right) \cos \left( 2\pi \frac{E_F}{\hbar \omega_c} \right), \] (6b)

where \( \sigma_0 = n_e e^2 \tau_s / m^* \). The temperature-dependent factor \( D(X) = X/\sinh(X) \) with \( X = 2\pi^2 k_B T / \hbar \omega_c \) describes the temperature damping of the oscillations. It can be derived by convoluting the DOS with the smoothed Fermi edge (Fig. 1) at finite temperatures.\(^{14}\)

In electrical measurements it is the resistivity \( \rho_{xx} \) that is detected. \( \rho_{xx} \) is obtained by inverting the conductivity tensor. If the scattering times are identified as above, the result is the simple expression\(^{12}\)

\[ \rho_{xx}(B) = \frac{1}{\sigma_0} \left[ 1 + 4D(X) \exp \left( -\frac{\pi}{\omega \xi} \right) \cos \left( 2\pi \frac{E_F}{\hbar \omega_c} \right) \right]. \] (7)

**B. Detection technique**

We use a Bruker ESP 300 EPR spectrometer, working at the microwave frequency of 9.5 GHz (\( X \) band). An Air Products, helium gas flow cryostat gives temperature control from LHe upward. The magnetic field covers the range from 0 to 1.4 T and is modulated by typical \( A = 6 \) G at 100 kHz, so that a lock-in technique can be used. Microwave power is usually 20 mW.

Samples of typical sizes between 1 and 10 mm\(^2\) are placed in the cavity with their surface perpendicular to the magnetic field and are cooled in the dark, usually to 4.2 K. At constant microwave frequency, the magnetic field is swept, causing an oscillatory behavior of the conductivity, which can be detected as changes in the reflected microwave power in the following way.

The microwave power that is reflected by a resonator, is determined by the ratio of the impedances of the waveguide and cavity.\(^{18}\) The impedance of the cavity is a function of the loss of electromagnetic waves in the walls of the cavity and in the sample. In a normal electron paramagnetic resonance (EPR) experiment, \( P_s \), the power loss due to the sample, has a resonant maximum when spin transitions extract energy from the magnetic part of the standing wave in the resonator, and transfer it to the lattice. In a SdH measurement, losses are due to the energy the conduction electrons extract from the electromagnetic wave when they are accelerated in the electric part of the field. Equation (8) describes the energy conservation in the cavity\(^{19}\)

\[ \frac{\partial w(r)}{\partial t} + \nabla \cdot \mathbf{S} + j(r) \mathbf{E}(r) = 0, \] (8)

where \( w(r,t) \) denotes the magnetic and electric field energy per unit volume, \( \mathbf{S} = \mathbf{E} \times \mathbf{H} \) is Poynting's vector, and \( j(r) \mathbf{E}(r) \) gives the ohmic losses due to the interaction of the field with the conduction electrons in the sample and the cavity wall. The losses in a sample of volume \( V_s \) are then given by

\[ P_s = \int_{V_s} j(r) \mathbf{E}(r) dV_s = \int_{V_s} (\sigma \mathbf{E}) \mathbf{E} dV, \] (9)

where \( \sigma \) is the two-dimensional conductivity tensor. Using \( \sigma_{xx} = \sigma_{yy} \) and \( \sigma_{xy} = -\sigma_{yx} \) in Eq. (9),

\[ P_S(B) = -\sigma_{xx}(B) \int_{V_s} E^2 dV_s. \] (10)

Taking current density and electric field as time dependent and complex and averaging the power loss over one cycle of the microwaves, leads to the equivalent result \( P_s(B) = \text{Re} \left\{ \sigma_{xx}(B) \right\} (E_0 E_0^\ast) dV_s, \) where \( E_0 \) and \( E_0^\ast \) denote the amplitude of the electric field and its complex conjugate, respectively. The power loss follows the oscillations of the conductivity with magnetic field. If the impedance of the cavity with sample properly matches the impedance of the waveguide, changes of the losses in the cavity cause proportional changes in the microwave power reflection.\(^{18}\) This is detected in a very sensitive way, using the field modulation and microwave bridge of the EPR spectrometer.

Two assumptions have been made. Firstly, the skin effect, which describes the finite penetration depth of a time-dependent electromagnetic field into a conductor, has been neglected. The skin length is defined by \( \delta = (2/\mu \omega \tau) \)\(^{1/2}. \) For a 100-A-wide quantum well (QW) or 2DEG, with a two-dimensional carrier concentration \( n_s = 10^{12} \text{ cm}^{-2} \) and a scattering time of 10 ps, \( \delta \) is as large as a tenth of a millimeter. Damping of the electric field can, therefore, be neglected in two-dimensional structures. The second approximation is to take the conductivity as independent of the microwave frequency. In the Drude approximation, the conductivity in a static magnetic field is given by \( \sigma = \sigma_0 [1 - i(\omega - \omega_0) \tau_s^0], \) with \( \omega = 0 \) in the static case. The assumption is therefore valid when \( \omega \tau \gg 1, \) i.e., for magnetic fields far above cyclotron resonance. In the \( X \) band, cyclotron resonance typically appears at \( B = 0.02 \text{ T}. \) At the same time, we need \( \omega \tau < 1, \) i.e., during one cycle of the microwave oscillation a large number of scattering events must take place. This, in the \( X \) band, is a good approximation only for transport scattering times smaller than 5 ps, which is the case in our samples.

An example of a SdH signal detected by microwaves from a modulation-doped, 120-Å-InGaAs/InP-single quantum well (sample 2, Sec. II D) is shown in Fig. 2. Two measurements with the sample in two different orientations to the magnetic field are shown. When \( B \) is applied perpendicular to the plane of the quantum well (lower curve) oscillatory behavior is clearly visible. With increasing angle between \( B \) and the normal to the surface, the oscillations disappear, leading to a cosine function (inset in Fig. 2), showing the two dimensionality of the effect.\(^{6}\) The strong background signal is probably due to magneto-conductivity effects, but has not been investigated in detail. Due to the modulation of the magnetic field, the first derivative of the signal is detected.
signals as detected with the magnetic field parallel ($\theta=90^\circ$, upper curve) and perpendicular ($\theta=0^\circ$, lower curve) to the surface, for a 120 Å InP/InGaAs quantum well (sample 2). The upper curve is drawn with an offset of 40 units. The first derivative of the signal is detected. The strong background signal must be removed for data analysis. The inset shows the cosine behavior of a peak position as a function of $\theta$. 

C. Data analysis

From SdH measurements one can extract the carrier concentration $n_{2d}$ and the effective mass $m^*$, as well as the single-particle relaxation time $\tau_s$. From Eq. (6a), it is readily observed that the frequency of the oscillation is $\nu = (E_F/E_{\text{HZ}}) B$. With the two-dimensional DOS, $g_0 = g_s g_v m^* / 2\pi \hbar^2$, and from $E_F = n_{2d}/g_0$ we obtain

$$n_{2d} = g_0 (e/\hbar) [\Delta (1/B)]^{-1}.$$  \hspace{1cm} (11)

Here $g_s$ and $g_v$ are the spin and valley degeneracy, respectively. Figure 3 is a plot of the peak number $n$ versus the inverse magnetic field for sample 1 (Sec. II D), showing the periodicity in $1/B$. The effective mass of the carriers can be deduced from the temperature dependence $A(T)$ of the amplitudes which is described by the factor $D(X)$ in Eqs. (6a) and (6b)

$$A(T) = \frac{T \sinh(\beta T m^*/Bm_0)}{T_0 \sinh(\beta T m^*/Bm_0)},$$  \hspace{1cm} (12)

with $\beta = 2\pi^2 k_B T_0 / e$. Figure 4 shows the temperature dependence of the amplitude in a modulation-doped, 150 Å InP/InGaAs single quantum well. The effective mass deduced gives evidence that the oscillation indeed originates in the quantum well (from Ref. 6). For an exact determination of the effective mass, high temperature stability and accurate absolute temperature determination are necessary.

To determine the single-particle relaxation time, the original data, shown in Fig. 2, are integrated, and the background is subtracted by a polynomial least square fit. The result is shown in Fig. 5. The behavior of the conductivity maxima in a magnetic field is

$$\Delta \sigma_{xx} \propto P(\omega_c \tau_s) D(X) \exp(-\pi/\omega_c \tau_s).$$  \hspace{1cm} (13)

FIG. 2. Signals as detected with the magnetic field parallel ($\theta=90^\circ$, upper curve) and perpendicular ($\theta=0^\circ$, lower curve) to the surface, for a 120 Å InP/InGaAs quantum well (sample 2). The upper curve is drawn with an offset of 40 units. The first derivative of the signal is detected. The strong background signal must be removed for data analysis. The inset shows the cosine behavior of a peak position as a function of $\theta$.

FIG. 3. SdH peak numbers as a function of inverse magnetic field. The slope of the linear fit gives the carrier concentration. The peak numbers are chosen in such a way, that the line goes through the origin. They then give the integer filling factor at each SdH maximum.

FIG. 4. Temperature dependence of the amplitude, $A(T)$, of the SdH signal in a 150 Å InGaAs/InP single quantum well (from Ref. 6). The solid line is the theoretical behavior for $m^* = 0.041 m_0$ (In$_{0.53}$Ga$_{0.47}$As), the dotted line for $m^* = 0.079 m_0$ (InP), showing that the signals originate in the In$_{0.5}$Ga$_{0.5}$As quantum well.

FIG. 5. Data from sample 2 as used for analysis. The original data (lower curve in Fig. 2) have been integrated, and the background removed by a polynomial fit.
It is dominated by the Dingle factor with \( \tau_s \) in the exponent. The prefactor \( P(\omega_c \tau_s) \) is equal to \( 4[\omega_c \tau_s/(1+\omega_c^2 \tau_s^2)]^2 \), according to Eq. (6a).

However, the prefactor should be treated with care. Our argumentation in Sec. II A, that identifies \( \sigma_{xx} \) as the measured property in a microwave-detected SdH measurement, is purely classical, and neglects, for instance, differences between ac and dc measurements, and the influence of the magnetic part of the microwaves on the result. Therefore, the correct prefactor might deviate slightly from the one given above. To keep the analysis of our data transparent, we simply assume the prefactor to be constant, i.e., we assume its influence on the shape of the oscillations to be small. With this assumption, we circumvent the problem that the transport scattering time \( \tau_s \) is usually unknown in a microwave measurement. Now \( \tau_s \) can be extracted from a plot of \( [\Delta \sigma_{xx}(B)/D(X)] \) on a logarithmic scale as a function of \( (1/B) \) (Dingle plot, Fig. 6). The slope of an exponential fit to the data points is then \( (-\pi m^*/e \tau_s) \). For the samples measured, the assumption of a constant prefactor leads to \( \tau_s \) values, which are roughly a factor of 2 larger than those obtained when using \( P(\omega_c \tau_s) \) as a prefactor. An experimental check for the correct prefactor would be the value of the extrapolated reduced conductivity \([\Delta \sigma_{xx}/\sigma_0 D(X)] P(\omega_c \tau_s)\) for \( 1/B=0 \), which should be equal to one according to Eq. (6a). This procedure would also offer information on the sample quality. However, this check cannot be performed, since no absolute values for \( \Delta \sigma_{xx} \) and, due to the modulation technique, no value for \( \sigma_0 \) can be obtained.

For the determination of \( \tau_s \) we have to face the difficulty that we need the effective mass as a parameter in \( D(X) \) and for the calculation of \( \tau_s \) from the slope. Usually, the bulk value from the tables is used, but in a two-dimensional system one has to use the in-plane effective mass of the carriers. Very recently it was shown, both experimentally and theoretically, for the case of InGaAs/InP quantum wells, that the in-plane effective mass increases rapidly for well thicknesses below 200 Å. This is mainly due to the non-parabolicity of the conduction band and the penetration of the electron wave function into the barrier. For this reason, it may be necessary to measure the effective masses, from the temperature dependence of the SdH oscillations, for example, or by using the cyclotron resonance technique (Sec. III A). This is especially important if \( \tau_s \) is measured as a function of the well thickness.

### D. Experimental results

SdH oscillations of good quality have been detected in lattice-matched, n-modulation-doped 2DEGs, like AlGaAs/GaAs or InGaAs/InP heterojunctions and strained-layer 2DEGs in AlGaAs/InGaAs heterojunction with a 700-Å-Si-doped layer (nominal 5 to 10×10^{17} cm^{-3}) and a 150 Å spacer grown on a Si-doped (5×10^{16} cm^{-3}) GaAs substrate. Sample 2 is a MOVPE sample, a one-sided, modulation-doped, 120 Å single quantum well of In_{0.53}Ga_{0.47}As, lattice matched to the surrounding InP, grown on a semi-insulating InP substrate. The 100 Å-thick n-doping layer (nominally 9×10^{17} cm^{-3}) is separated from the quantum well by a 80 Å spacer layer. The mobility is 60 000 cm^{2}/V s (77 K).

Microwave-detected SdH oscillations in sample 1, measured at \( T=4.2 \) K, exhibit the single-particle relaxation time \( \tau_s=0.6 \) ps in the magnetic field range 0.6 to 1.4 T, when the prefactor in Eq. (13) is taken as constant. For the effective mass, \( m^*_s/\varepsilon r_0 = 0.067 \) is used. From the slope of the peak locations (Fig. 3) and with Eq. (11), the carrier concentration is determined to be \( n_{2d}=7.1×10^{11} \) cm^{-2}. For comparison, electrical measurement have been performed on a contacted piece of the wafer. A Hall mobility of 95 000 cm^{2}/V s at \( T=4.2 \) K is obtained, corresponding to a transport scattering time \( \tau_s=3.6 \) ps. From the measurement of the SdH oscillations (\( T=4.2 \) K) in resistivity mode and an analysis according to Eq. (7), we get the single-particle relaxation time \( \tau_1=0.5 \) ps (\( \tau_1/\tau_s=7.2 \)) and for the carrier concentration \( n_{2d}=6.5×10^{11} \) cm^{-2}. Even though the measurements have been performed on different pieces of the wafer, the agreement between the results obtained with the two techniques (Table I) is reasonable. The first oscillation could be resolved at about 0.8 T, or

**TABLE I. Results for sample 1.**

<table>
<thead>
<tr>
<th>Sample</th>
<th>Microwave</th>
<th>Electrically detected SdH</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_s ) (ps)</td>
<td>0.6</td>
<td>0.5</td>
</tr>
<tr>
<td>( \tau_1 ) (ps)</td>
<td>...</td>
<td>3.6</td>
</tr>
<tr>
<td>( n_{2d} ) (cm^{-2})</td>
<td>7.1×10^{11}</td>
<td>6.5×10^{11}</td>
</tr>
</tbody>
</table>
ωτ₁ ≈ 1, which should be compared to 0.6 T in the case of SdH oscillations detected by microwaves. The sensitivity of the two techniques is comparable.

Using the prefactor \( P(ω, τ) \) instead of the constant in the analysis of the data from the microwave detection, the value \( τ_1 = 0.3 \) ps is obtained. This is, however, less close to the result from the electrical measurement.

From a fit of Eq. (13) to the Dingle plot in Fig. 6 we determine the single-particle relaxation time in sample 2 to be \( τ_1 = 0.23 \) ps at \( B = 1 \) T (prefactor assumed constant). For the effective mass, the value \( m^*_e = 0.051 \) \( m_0 \) has been used, as obtained from cyclotron resonance measurements (Sec. III A). The Dingle plot shows a curvature, apparently corresponding to a linear to parabolic increase in \( τ_e \) with magnetic field. Figure 7 shows \( τ_e(B) \), deduced from the derivative of ln[\( σ_{xx}(B)/D(X) \)]. A behavior like this can be due to inhomogeneities in the quantum well on a scale larger than the cyclotron radius. On the other hand, it might also be an artifact, caused by the detection technique or the data analysis. The carrier concentration is determined to be \( 1.04 \times 10^{12} \) cm\(^{-2} \), corresponding to about 50% occupation of the first subband. A Fourier transformation shows only one frequency, confirming the second subband to be unoccupied.

III. COMPARISON WITH OTHER CONTACT-FREE CHARACTERIZATION TECHNIQUES

After having compared the microwave detection technique of SdH oscillations to electrical measurements, we report in this section on the application of other contact-free methods on sample 2. For comparison, values for the single-particle relaxation time are determined. The measurement of the effective mass, necessary for the analysis of the SdH data, is described. An overview of the results is given in Table II.

A. Cyclotron resonance

A conventional, contact-free method for semiconductor characterization is cyclotron resonance (CR). A sample in a magnetic field absorbs microwave or far infrared (FIR) quanta, when their energy equals the energy spacing between the Landau levels. From the shape and the position of the resonance line, the single-particle relaxation time and the effective mass, \( m^* \), respectively, can be determined.

The absorption of electromagnetic waves in a two-dimensional layer is given by the real part, \( σ_R(ω) \), of the dynamic conductivity, which can be approximated by

\[
σ_R(ω) = \frac{e^2 n^{2d}}{m^*} \frac{(Γ/2)}{(ω - ω_c)^2 + (Γ/2)^2},
\]

where \( Γ \) is the linewidth [full width at half maximum (FWHM)] in frequency units. At resonance, transitions take place between the last occupied and the first unoccupied Landau level at the Fermi level, so that \( Γ \) is obviously determined by the broadening of the Landau levels at Fermi energy, and thus by the single-particle relaxation time. So far no generally valid expression for the relation between \( Γ \) and \( τ_e \) has been presented.\(^{16}\) Approximately, we can use (Refs. 26 and 27) \( Γ = \Gamma_c = 1/τ_c \), where \( Γ_c \approx Γ_{n+1} = 1/τ_{n+1} \). Replacing \( Γ_c \) in Eq. (14) by \( 1/τ_e \), we can determine \( τ_e \) by a fit of Eq. (14) to the resonance line. The resonance field \( B_0 \) gives the effective mass by the relationship \( ω_c = eB_0/m^* \).

We performed a FIR-CR measurement\(^{28} \) on sample 2. The light of a CO\(_2\)-pumped, FIR laser (Edinburgh) \( 20 \) mW was mechanically chopped and guided via a light guide to the sample, which was placed in a 8 T magnet. The transmission of the FIR light was measured with an Allen-Bradley resistor, showing a minimum at resonance (Fig. 8). The chopping frequency was 17 Hz and the temperature of the sample 6 K. The FIR lines used were 118.83 and 163.03 μm.

From the estimated position of the resonance minima, we determine the effective mass to be \( m^*_e = 0.051 \) ± 0.001 \( m_0 \) which is well above the table value of \( m^*_e = 0.041 \) \( m_0 \) for In\(_{0.53}\)Ga\(_{0.47}\)As. From a fit of the line

| TABLE II. Results for sample 2, obtained using different contact-free techniques. |
|---------------------------------|-----------------|-----------------|-----------------|-----------------|
|                                | Microwave       | CR              | ODCR            | Magneto-PL      |
| \( τ_e \) (ps)                 | 0.23 (\( B = 1 \) T) | 0.85 (\( B = 3.4 \) T) | \( \cdots \)     | \( \cdots \)     |
| \( n_{2d} \) (cm\(^{-2} \))  | \( 1.04 \times 10^{12} \) | \( \cdots \)     | \( 1.32 (\( B = 4.6 \) T) \) | \( 1.0 \times 10^{12} \) |
| \( m^*_e (m_0) \)             | \( \cdots \)     | 0.051           | 0.051           | \( \cdots \)     |
shape to a Lorentzian, we obtain, using Eq. (14) $\tau_s=1.22$ ps at $B=4.6$ T with the 118.8 $\mu$m line, and $\tau_s=0.85$ ps at $B=3.4$ T with $\lambda=163.0$ $\mu$m (Table II). Although, for the reasons mentioned above, the values are difficult to compare with those obtained by SdH measurements, the increase of the single-particle relaxation time as observed in SdH oscillations detected by microwaves can apparently be confirmed (Fig. 7). The shape of the resonance is asymmetric. This can be explained by interference effects of the FIR light in the sample.24

C. Magneto photoluminescence

B. Optically detected cyclotron resonance

In undoped, two-dimensional structures, the total number of carriers is often too low to make the detection of cyclotron resonance possible. The sensitivity can be strongly enhanced by optical detection of the CR (ODCR).21,28-30

In an ODCR experiment, changes in the photoluminescence (PL) of a sample induced by microwave or far infrared illumination are monitored as a function of the magnetic field. Under cyclotron resonance conditions, the free electrons gain energy, and the PL is affected by heating effects or by the impact ionization of bound electrons or excitons. When the PL is observed at fixed energy, while the magnetic field is swept, CR is detected very sensitively as an increase or decrease in PL intensity.

For sample 2, we performed an ODCR experiment, using the same setup as for the classical FIR-CR measurement. In addition to the 118.8 $\mu$m FIR line, the sample was illuminated with the 514.5 nm line of a 6 mW argon ion laser via an optical fiber. The PL signal was collected via the same fiber, dispersed by a 0.22-m-focal-length single monochromator, and detected with a cooled Ge detector. To allow for the lock-in technique, the FIR light was mechanically chopped at 17 Hz.

Figure 9 shows the ODCR spectrum at the PL energy of 0.85 eV. Fitting again to Eq. (14), we find, in agreement with the CR $m^* = 0.051 \pm 0.001 m_0$ and the slightly higher single-particle relaxation time, $\tau_s=1.32$ ps. This might be an observation of the photoneutralization of impurities, which is often reported to take place in optical detection of CR.31 Furthermore, the asymmetry in the line shape has changed, probably due to a change in the carrier concentration, and thus the refraction index, in the quantum well induced by the illumination.24

Another contact-free characterization technique that we applied on sample 2 for the comparison with the microwave detection of SdH oscillations, is photoluminescence (PL) in a magnetic field (magneto-PL). The solid line in Fig. 10 is a photoluminescence spectrum of the InP/InGaAs quantum well (sample 2) at zero magnetic field, measured in the same setup as the ODCR experiments, but without FIR illumination. We see an onset of the intensity at about 825 meV, increasing intensity.

Figure 9 shows the ODCR spectrum at the PL energy of 0.85 eV. Fitting again to Eq. (14), we find, in agreement
Two Landau series are identified, the upper one ascribed to direct, free electron to localized hole transitions (Fig. 12). The PL as a function of the magnetic field (inset, constant detection energy $E_{ph}=0.87$ eV) has a maximum every time a Landau level coincides with the detection energy (Landau oscillations). $E_F$ is the estimated position of the Fermi energy at zero magnetic field.

They explained the unusual line shape (Fermi edge enhancement) to be caused by the localization of the holes at high energies and a drop at about 870 meV. Instead of this, for a degenerately doped, direct-band-gap, semiconductor quantum well, one would expect a sharp exponential decrease of the hole occupation for transitions for electrons up to the Fermi energy to take place. The enhancement of the intensity is explained by electron-hole Coulomb interaction. This anomalous behavior is an advantage for studies of the electron DOS, because in contrast to the modeled ideal quantum well, electrons from all occupied states in the conduction band dominate the spatially uniform intensity.

The dotted spectrum in Fig. 10 is measured with a 4.5 T magnetic field applied perpendicular to the surface of the sample. Five peaks are resolved, their number and position change with magnetic field, motivating the assumption, that the spectrum reflects the density of states. In a plot of the peak positions versus magnetic field (Fig. 11) two Landau fans can be identified. The origin of the transitions is modeled in Fig. 12. The holes are assumed to lie close to the valence band maximum on localized states which are spread in $k$ space. Direct transitions, denoted by $n-h$, from the electron Landau levels to the hole state give rise to the upper Landau series in Fig. 11. The energy position of Landau level $n$ is given by $E_n(B)=E_0+\hbar\omega_c(n+1/2)$. From the intercept at $B=0$ of linear fits to the peak positions we determine $E_0=826\pm1$ meV. The lower series $n-A$ is possibly due to electron-acceptor transitions to an acceptor located at about 31 meV above the hole subband, but has, to the best of our knowledge, not been identified.

The slope of the peak energies versus magnetic field is determined by the electron effective mass, but can be reduced by the downward bound dispersion function of the localized holes (Fig. 12). We find that $m^*_e=0.045\pm0.002m_0$, which is significantly lower than the CR effective mass (Table II). The spacing $\hbar\omega_c$ between different peaks at constant magnetic field leads to the same value. Interpreting this mass as the reduced mass of electrons of $m^*_e=0.051\pm0.001m_0$ and the localized holes, we deduce $m^*_h=0.38\pm0.15m_0\approx7.5m^*_e$, which is surprisingly close to the table value of 0.377 for the free heavy hole.

If the PL intensity is observed at fixed energy, $E_{ph}$, while the magnetic field is swept, Landau oscillations periodic in $1/B$ are obtained (inset in Fig. 11). Even though the oscillations look very similar to SdH oscillations, interpretation must be handled with great care. The period of the oscillation is, according to Eq. (5) given by $f=(E_{ph}-E_0)/\hbar\omega_c$, and does not give the carrier concentration $n$ if $E_{ph}-E_0$ is different from $E_F$. The carrier concentration could be obtained by measuring the period of the Landau oscillations close to $E_F$, e.g., in the intensity of an exciton, or by measuring the intensity of the 0-0 transition which is dependent on the magnetic field, and oscillates with the DOS at the Fermi level $g_0(E_F)$ due to changing screening of the photoexcited holes. From the energy difference between $E_0$ and the highest resolved peaks at low fields, where the oscillation amplitude of the Fermi level is small, we estimate $E_F(1+m^*_e/m^*_h)=54\pm3$ meV. The factor $(1+m^*_e/m^*_h)$ is necessary because of the downward bound hole dispersion function. With the DOS $g_0=m^*/\pi\hbar^2$, $m^*_h=0.051m_0$, and $m^*_e=0.38m_0$, the carrier concentration can be estimated to be $n_d=1.0\pm0.1\times10^{12}$ cm$^{-2}$ compared to $1.04\times10^{12}$ cm$^{-2}$ obtained from the SdH oscillations (Table II). The shape of the oscillations might be expected to be determined by the single-particle relaxation time according to Eq. (5). However, a Dingle plot of intensity versus inverse magnetic field exhibits an only slight increase in $\tau_s$ (Fig. 7) where $\tau_s\approx90$ fs. An estimation of...
the Landau level broadening $\Gamma = \hbar / \tau = 6$ meV in the PL spectrum leads to about $\tau_c \approx 120$ fs. Both values are much lower than those obtained by the CR and SdH measurements (Fig. 7, Table II) at the same magnetic field. It is, thus, clear that the broadening of the peaks in PL is not dominated by the broadening of the Landau levels. If we assume that the broadening in PL is due to an uncertainty in the hole $k$-space, $\Delta k$, corresponding by $\Delta k \Delta k > 1/2$ to a localization radius $\Delta r$, we obtain, with $m^* = 0.3 m_0$, the value $\Delta r > 8 \AA$. This agrees with a value of 10 to 30 Å reported by Skolnick et al.\textsuperscript{34} who deduced it from the magnitude of the exciton–phonon coupling, and with theoretical investigations of the Fermi edge enhancement by Rorison.\textsuperscript{36} Therefore, in samples with hole localization, $\tau_c$ cannot be found by PL Landau oscillations. Samples, that do not show Fermi edge enhancement however, will usually not give oscillations of the required quality because of the low intensity at higher PL energies.\textsuperscript{35} For these reasons, PL Landau oscillations seem not to be appropriate for the detection of the single-particle relaxation time.

IV. DISCUSSION

The new technique of microwave detection of Shubnikov–de Haas oscillations has been applied to a variety of samples containing two-dimensional electron systems, without finding restrictions in its applicability. The precise determination of carrier concentrations and, with sufficient temperature stability, effective masses is possible. The single-particle relaxation time, $\tau_c$ can be determined from the shape of the oscillations. The proposed data analysis procedure overestimates $\tau_c$ possibly by a factor of 2. Drawing conclusions concerning carrier mobility from $\tau_c$, is not straightforward.

Comparison with electrical measurements of the SdH effect shows that the results agree (Table I). In contrast to microwave measurements, electrical measurements of the resistivity make direct determination of the mobility possible. The sensitivities of both techniques are comparable, while the destructive and often complicated contacting procedure is not necessary for the microwave detection technique. Therefore, this latter kind of measurement is easier to prepare and, in addition, the risk of affecting the signal by not used contacts is avoided.

Other contact-free characterization techniques, providing this same advantage, have been applied to the same sample 2. By cyclotron resonance, the effective mass has been determined, which is very accurate and in a way easier than by a measurement of the SdH effect, because the demands to the cryostat are not so high. From the broadening of the CR resonance line, $\tau_c$ was determined. It shows rough agreement with the SdH measurement when the behavior with increasing magnetic field is taken into account. However, because there exists no general, valid relation between $\Gamma_c$ and $\tau_c$, values for $\tau_c$ obtained from CR must be treated with care. The sensitivity of the CR technique can be improved by optical detection, but even this technique failed for quantum wells thinner than 50 Å, while microwave SdH measurements were successful down to 30 Å.

For sample 2, the detection of Landau oscillations was possible in PL, too. This is due to Fermi edge enhancement, caused by hole localization. But for the same reason, the broadening of the observed PL peaks is not determined by $\tau_c$ and values obtained for the effective mass are misleading. In samples without Fermi edge enhancement, the determination of the effective masses of electrons and holes would be possible,\textsuperscript{22} but Landau oscillations in PL will usually not be detectable and of sufficiently high quality to allow the determination of $\tau_c$. Carrier concentrations can be obtained by magneto-PL in any case, but, in comparison with SdH measurements, the necessary experimental efforts are considerably greater.

V. SUMMARY

In summary, the technique of microwave detection of Shubnikov–de Haas oscillations has been demonstrated and compared to electrical measurements. Some uncertainties in data analysis have to be balanced to the practical advantages of a contact-free method. The other contact-free characterization techniques applied in this work can compete in certain cases, but do not have the same range of applicability.

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Another way out is to express the results for \( \tau \) normalized to \( m_f \), e.g., as an "equivalent mobility" \( \mu_e = \left( \frac{\tau \mu}{m_f} \right) \). The temperature factor \( D(T) \) can be approximated as \( \frac{1}{2} \sqrt{\frac{\pi}{6T}} \), which is valid for \( T > 4.2 \text{ K} \). Temperature broadening and collision broadening are additive and scale with the effective mass in the same way. It is \( m_f^2 / \tau \) that determines the collision broadening.

For sufficiently short range scatterers, where \( \tau = \tau_f \) (Ando, see Refs. 8 and 9) derives the broadening independence of the level as \( \Gamma_f / \hbar = (2\omega_f \tau_f)^{1/2} \), where \( \tau_f \) is the scattering time at zero field. A roughly correct behavior \( \Gamma_f \propto (B/\tau_f)^{1/2} \) was observed by Abstreiter (see Ref. 24) in surface inversion layers on Si. In our samples however, the condition \( \tau_f = \tau_r \) is not fulfilled by far. In general, the Landau level (LL) broadening \( \Gamma = \hbar / \tau_r \) is a function of both the LL index and the magnetic field (see Ref. 9). Values for \( \tau_r \), obtained by cyclotron resonance, must be treated with care.

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