Math 4820 Lecture 1

January 26, 2016

Outline

• Policies
• Prerequisites
• Syllabus
• Definitions for scientific computing
• Review of ODEs

First Half

Course Policies (Homework, grading, etc)
Prerequisites

Students taking this class should be familiar with the material in:

- MATH 2100 (introductory multivariable calculus and linear algebra)
- MATH 2400 (introductory ODEs and PDEs)

The additional courses are not strictly necessary (there is only a little overlap) but are still recommended:

- MATH 4100 (a second course in linear algebra)
- MATH 4800 (NUMCOM)

Some additional topics that we will use:

- Fourier transforms for von Neumann stability analysis
- complex numbers
- Ability to write small programs ($O(100)$ lines)
- unit tests
- Knowledge of Python or MATLAB
Syllabus

Second Half

Definitions for Scientific Computing

Accuracy

Computational Cost

Memory Requirements

Well Posedness (Hadamard)

Convergence
Existence and Uniqueness

Linear Problems

By Duhamel’s principle, if we can write a system of ODEs as
\[ \ddot{u}(t) = A\dot{u}(t) + \vec{g}(t) \]
then the solution (for a given initial condition \( \vec{u}_0 \)) is
\[ \vec{u}(t) = \exp(A(t-t_0))\vec{u}_0 + \int_{t_0}^{t} \exp(A(t-\tau))\vec{g}(\tau) \, d\tau. \]

Put another way, all linear problems have unique solutions.

Nonlinear Problems

Existence of a solution to a nonlinear problem is harder: it requires that the function is Lipschitz continuous over the domain of interest:

In particular, if \( u'(t) = f(t) \) and \( f \) is Lipschitz continuous in the domain \( \mathcal{D} \), then:

An Example

Consider \( u'(t) = \sqrt{u(t)} \), \( u(0) = 0 \). Does this problem have a unique solution?
Characterization of ODE Behavior
Most of the analysis in this course is done on linear ODEs. In particular, we will use the model equation

\[ y' = \lambda y \]

for complex valued \( \lambda \). (We could also use \( y' = \lambda y + g(t) \), but the results would be the same).
Systems of ODEs

Example

Rewrite

\[ y''' + ay'' + by' + cy = g(t) \]

as a first order system of ODEs.
Example (a preview of hyperbolic problems)

Rewrite

\[ u_{tt} = c^2 u_{xx} \]

as a system of first order PDEs.