Math 4100 Lecture 1

September 1, 2015

Administrative Details

Topics for Today

First Half

• Course information
• syllabus
• Vectors, matrices, and linear combinations
• Homework 1: due Friday

Second Half

• Lengths, angles, and dot products

First Half

Course Information

• I will post announcements, assignments, and grades through Blackboard.
• The best way to get a hold of me is by email: wells2@rpi.edu.
• My office is in Amos Eaton 311. My office hours are not yet set.

Vectors and Linear Combinations

Example

As motivation, how many ways may we interpret

\[ 2x - y = 0 \]
\[ -x + 2y = 1? \]
Arithmetic with Vectors

Addition

Scalar multiplication

Linear Combination of Vectors

Definition
Solution by Geometric Methods

Consider the first example again. When, geometrically, does a system of two equations have:

- No solution,
- One solution, or
- Infinitely many solutions?

The First Example Again

Consider the first example again:

\[ 2x - y = b_1 \]
\[ -x + 2y = b_2 \]

May we solve this system for arbitrary \( b_1 \) and \( b_2 \)? Why or why not?
Example

Consider

\[ \vec{u} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \vec{v} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}. \]

What can we make out of linear combinations of \( \vec{u} \)? How about \( \vec{u} \) combined with \( \vec{v} \)? How about \( \vec{u} \) and \( 2\vec{u} \)?

Example

What do linear combinations of the set of vectors

\[ \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \right\} \]

describe?
Another instance of the first example

How can we rewrite the first example,

\[
\begin{align*}
2x - y &= 0 \\
-x + 2y &= 1,
\end{align*}
\]

with dot products?
Dot Products, Norms, and Length

The dot product of a vector with itself looks like the Pythagorean theorem:

Angles between vectors

We know that two vectors in 2D or 3D define a plane. How can we calculate the angle between two vectors that are in the same plane?
The Cauchy-Schwarz Inequality

From the cosine formula we have the most important inequality in linear algebra:

The Triangle Inequality

Some Examples

Calculate the angle between $\left( \begin{array}{c} 1 \\ \sqrt{3} \end{array} \right)$ and $\left( \begin{array}{c} 1 \\ 0 \end{array} \right)$

and verify the Cauchy-Schwarz and Triangle inequalities.
Problem 8
This problem is out of the book. Determine whether or not the following statements are true:

Part (a)
If \( \vec{u} \) is perpendicular (in three dimensions) to \( \vec{v} \) and \( \vec{w} \), then \( \vec{v} \) and \( \vec{w} \) are parallel.

Part (b)
If \( \vec{u} \) is perpendicular to \( \vec{v} \) and \( \vec{w} \), then \( \vec{u} \) is perpendicular to \( \vec{v} + 2\vec{w} \).

Part (c)
If \( \vec{u} \) and \( \vec{v} \) are perpendicular unit vectors then \( \| \vec{u} - \vec{v} \| = \sqrt{2} \).