Math 4100 Homework 9

Due: 8:00 AM, December 4

(2 pts) Problem 1

Consider the linear mapping $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined on the Cartesian basis as

$$
T \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ 0 \end{pmatrix},
$$

$$
T \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 3 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 4 \begin{pmatrix} 0 \\ 1 \end{pmatrix}.
$$

Find bases for the domain and range of $T$ so that the matrix of the transformation between those bases is diagonal.
Problem 2

Find the pseudoinverse of

\[
\begin{pmatrix}
1 & -1 \\
-1 & 1
\end{pmatrix}.
\]
(2 pts) Problem 3

Diagonalize the skew-Hermitian matrix to find $K = UΛU^H$. What are the eigenvalues?

$$K = \begin{pmatrix} 0 & -1 + i \\ 1 + i & i \end{pmatrix}$$
(2 pts) Problem 4

The Gram-Schmidt process changes a basis $\vec{a}_1, \vec{a}_2, \vec{a}_3$ to an orthonormal basis $\vec{q}_1, \vec{q}_2, \vec{q}_3$. These are columns in $A = QR$. Show that $R$ is the change of basis matrix from the $\vec{a}$s to the $\vec{q}$s. 

Hint: $\vec{a}_2$ is which combination of $\vec{q}$s when $A = QR$?