Problem 1

Part (a)
Find the determinant of the rank one matrix

\[
A = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \begin{pmatrix} 1 & -4 & 5 \end{pmatrix}
\]

.

Part (b)
Find the determinant of the skew-symmetric matrix

\[
\begin{pmatrix} 0 & 1 & 3 \\ -1 & 0 & 4 \\ -3 & -4 & 0 \end{pmatrix}
\]
Problem 2

Part (a)

Use row operations to find the determinant of

\[
\begin{pmatrix}
0 & a & 0 & 0 \\
0 & 0 & b & 0 \\
0 & 0 & 0 & c \\
d & 0 & 0 & 0
\end{pmatrix}.
\]

Part (b)

The Hadamard matrix $H$ has orthogonal rows. Explain (without computing anything) what $|\det(H)|$ must be, where

\[
H = \begin{pmatrix}
1 & 1 & 1 & 1 \\
1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1 \\
1 & -1 & 1 & -1
\end{pmatrix}.
\]
Problem 3

Find the eigenvalues and eigenvectors of
\[
\begin{pmatrix}
2 & 2 & 2 \\
2 & 2 & 2 \\
2 & 2 & 2 \\
\end{pmatrix}.
\]

What are their multiplicities?
Problem 4

Diagonalize $B$ to prove the formula

$$B^k = \begin{pmatrix} 5 & 1 \\ 0 & 4 \end{pmatrix}^k = \begin{pmatrix} 5^k & 5^k - 4^k \\ 0 & 4^k \end{pmatrix}.$$