Problem 1

(1 pt) Part (a)
If $A^T y = d$ has a solution, then in which of the four subspaces does $d$ belong?

(1 pt) Part (b)
Let $A = uv^T$ be a $2 \times 2$ matrix with rank 1. What are its four fundamental subspaces?
(2 pts) Problem 2

Describe the four subspaces of \( \mathbb{R}^3 \) associated with

\[
A = \begin{pmatrix}
1 & 1 & 0 \\
0 & 1 & 1 \\
1 & 1 & 1
\end{pmatrix}.
\]
(2 pts) Problem 3

Assume $A$ is a symmetric matrix in parts (a) and (b) below.

(1 pt) Part (a)

Explain why the column space of $A$ is perpendicular to the null space of $A$.

(1 pt) Part (b)

If $A\vec{x} = \vec{0}$ and $A\vec{z} = 5\vec{z}$, which subspaces contain these “eigenvectors” $\vec{x}$ and $\vec{z}$?
(2 pts) Problem 4

Suppose $A$ is the $4 \times 4$ identity matrix with its last column removed (so $A$ is $4 \times 3$). Project $\vec{b} = (1, 2, 3, 4)$ onto the column space of $A$. What is the shape of the projection matrix $P$ and what is $P$?