Math 4100 Homework 1

Due: 8:00 AM, September 11

(2 pts) Problem 1

Let $\vec{u}$ and $\vec{v}$ be two orthogonal vectors in $\mathbb{R}^n$. Formulate and prove a version of the Pythagorean theorem that applies to $\vec{u}$ and $\vec{v}$. 
(2 pts) Problem 2

Find two vectors $\vec{v}$ and $\vec{w}$ that are perpendicular to $(1,0,1)$ and to each other.

(2 pts) Problem 3

Find two different combinations of the three vectors

$$\vec{u} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}, \quad \vec{v} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}, \quad \text{and} \quad \vec{w} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$$

that produce

$$\vec{b} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$ 

For any three vectors in the plane, will there always be at least two different combinations that produce $\vec{b}$?
(2 pts) Problem 4

Find the pivots and the solutions for $A\vec{x} = \vec{b}$, where

$$A = \begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix} \quad \text{and} \quad \vec{b} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 5 \end{pmatrix}. $$