Sampling-induced hidden cycles in correlated random rough surfaces

H.-N. Yang,* Y.-P. Zhao, A. Chan,† T.-M. Lu, and G.-C. Wang

Department of Physics, Applied Physics, and Astronomy and Center for Integrated Electronics and Electronics Manufacturing, Rensselaer Polytechnic Institute, Troy, New York 12180-3590

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We show both experimentally and theoretically that the sampling-induced hidden cycles can exist in scale-invariant rough surfaces having a correlation length \( \xi \). If the sampling size \( L \) is sufficiently large, the oscillatory behavior will diminish with the fluctuation within an order of \( (\xi/L)^{1/2} \). This is consistent with the law of large numbers for the correlated systems: the average of \( N \)-correlated variables having a correlation length \( \xi \) will converge to their mean within an order of \( \sqrt{\xi^2/N} \). Based on this result, we propose that in order to distinguish the mound surface from the self-affine surface, the sampling condition \( \sqrt{\xi^2/N} \ll 1 \) and an average of a large number of images are required. [S0163-1829(97)01631-7]

I. INTRODUCTION

The strong law of large numbers is probably the most well-known and the most useful theorem in computing statistical quantities.\(^1\) It states that the average of \( N \)-independent random variables having the same distribution will converge to the mean of that distribution within an order of \( 1/\sqrt{N} \). Practically, this theorem allows one to estimate an average quantity by simply computing its sampled mean.

However, the law of large numbers might not be applicable to a system in which the random variables are not independent. One example is a rough surface topography where the surface heights \( z(r) \) are not completely independent within certain distance. Here, \( r = (x, y) \) denotes the lateral coordinates of a surface position. The correlation between two surface positions separated by a distance \( r \) is usually described by the height-height correlation function,

\[
G(r) = \langle (z(r) - z(0))^2 \rangle.
\]  

(1)

If \( z(r) \) and \( z(0) \) are independent, \( G(r) = 2w^2 = \text{const} \), where \( w \) is the interface width of the surface defined as

\[
w^2 = \langle (z(r) - z(r))^2 \rangle.
\]  

(2)

However, if \( z(r) \) and \( z(0) \) are not independent, the height-height correlation function \( G(r) \) will be a function of \( r \), instead of a constant, \( 2w^2 \). One type of correlated rough surface is the scale-invariant rough surface \( \xi^2 \) which has a correlation relation

\[
G(r) = \begin{cases} 
\xi^2 & \text{for } r \leqslant \xi \\
2w^2 & \text{for } r \geqslant \xi,
\end{cases}
\]  

(3)

where \( \xi \) is the lateral correlation length, a distance within which the surface height fluctuations are correlated and dependent, but beyond which the variations spread and are not correlated (independent). The scale-invariant behavior in this type of rough surface is shown in the correlated region, \( r \leqslant \xi \), as characterized by a power-law form, \( G(r) \sim r^{2\alpha} \). The exponent \( \alpha \) describes surface roughness and is limited within the range \( 0 \leqslant \alpha \leqslant 1 \). A plot of a typical height-height correlation function for this type of rough surface is shown in Fig. 1(a). For \( \alpha < 1 \), the surface morphology is a self-affine fractal with an “anisotropic” scaling relationship between the vertical and lateral directions. For \( \alpha = 1 \), since the scaling relationship becomes isotropic, the corresponding morphology is not self-affine, but self-similar.

Another type of correlated rough surface is the morphology of a collection of fairly regular three-dimensional mounds,\(^6–12\) characterized by a well-defined separation distance \( \lambda \). This type of surface is not self-affine. A typical plot of its height-height correlation function is shown in Fig. 1(b). Note that the distinct difference between the scaling surface

![Fig. 1. The height-height correlation function from two types of correlated rough surfaces: (a) The scale-invariant rough surfaces; (b) the surfaces having regular mound structures.](image-url)
morphology [Fig. 1(a)] and the mound surface structure is the existence of the oscillatory behavior at \( r > \lambda \), shown in Fig. 1(b), where the oscillation period is related to the mound separation \( \lambda \). The scaling rough surfaces do not have such oscillatory behavior because their roughness features are more random and do not have fairly periodic cycles (mounds) described by a single length scale \( \lambda \).

As we pointed out earlier, as far as the statistical behavior of a correlated system is concerned, the law of large numbers has to be modified. For example, the convergence of the sampled \( w \) and \( G(r) \) to their true values will not follow the same rule of \( 1/\sqrt{N} \) as in the case of independent variable system. Obviously, the law should be modified as \( \sqrt{\bar{e}^2/N} \), where \( d \) denotes the dimensionality and \( \bar{e}^2/N \) is the number of the independent cells in the correlated system.

Another important issue for the correlated system is the search for periodic cycles that might be hidden by noises or irregular perturbations. In the case of correlated rough surfaces discussed above, the cycles do exist in the mound structures, but do not exist in the scaling surfaces. To determine whether or not such cycles exist is important, for it allows one to further determine the driving mechanism for the formation of the corresponding morphologies. For example, a scaling rough surface can be generated by the fluctuation during thin-film growth where, due to the competition between fluctuation and relaxation, the growing surface exhibits scaling characteristics. On the other hand, the mound structures can be formed in molecular-beam-epitaxy growth, where the step-edge diffusion barriers (Schwöbel barriers\(^{13} \)) prevent the landing atoms from migrating downward. To distinguish these two mechanisms, one may rely on whether or not the oscillation exists in \( G(r) \) as obtained from the measured surface images. However, we will show later that the analysis might not give a correct answer due to the hidden cycles induced by sampling practice.

In this paper, we present our study on the statistical analysis of the correlated scaling rough surfaces. We shall show both experimentally and theoretically that the sampling-induced hidden cycles do exist in the scale-invariant rough surfaces. Such oscillatory behavior will diminish when the sampling size is sufficiently large. The oscillation amplitude approaches zero to within an order of \( \sqrt{[(L/L)^d]} \), where \( L \) is the surface sampling size and \( d \) is the surface dimensionality \( (d = 1, 2) \). Both \( \xi \) and \( L \) are in units of the spacing of neighboring data points. This is consistent with the law of large numbers for the correlated systems: the average of \( N \)-correlated variables having a correlation length \( \xi \) will converge to their mean within an order of \( \sqrt{\bar{e}^2/N} \).

The outline of this paper is the following: Sec. II shows the analysis on two surface topographies measured from atomic force microscopy (AFM) and scanning tunneling microscopy (STM), respectively. Section III focuses on a solvable scale-invariant model surface, where the analytical results show the existence of sampling-induced hidden cycles and the modified law of large numbers for the correlated surface. A brief discussion will be given in Sec. IV.

**II. ANALYSIS OF THE SURFACE IMAGES MEASURED FROM AFM AND STM**

In the imaging techniques such as AFM and STM, the image scan scale, \( L_x \times L_y \), determines the sampling size for the statistical analysis. Assume that \( a \) is the spacing of neighboring data points along the \( x \) or \( y \) directions. For simplicity, in this paper, we define all the length scales to be in units of \( a \). Thus, we have \( L_x = N_x a \) and \( L_y = N_y a \), where \( N_x \) and \( N_y \) are the number of data points along the \( x \) and \( y \) directions, respectively. In practice, for an image measured from AFM or STM, the statistical average must consider the tip scan direction. The noise influence on the measurement is much less significant along the fast scan direction (defined as the \( x \) direction) than that along the slow scan direction (defined as the \( y \) direction). The realistic calculation of \( G(r) \) from the sampling of discrete raw data should then be averaged along the \( x \) direction as\(^{15} \)

\[
G(r) = \frac{1}{N_x(N_x-m)} \sum_{n=1}^{N_x-m} \sum_{l=1}^{N_y} [z(x+n,l) - z(n,l)]^2
\]

where \( x = m \). Note that we use the symbol \( G_S(x) \) to represent the experimental average, i.e., the sampling height-height correlation, in order to distinguish it from the true value, \( G(x) \). In Eq. (4), the number of terms to be averaged is \( N_x(N_x-m) \). Usually, we restrict ourselves in the case of \( N_x \gg m \), which can guarantee sufficient statistics for the average. Thus, the number of terms averaged is approximately \( N_xN_x \). With the algorithm given by Eq. (4), we are able to analyze the height-height correlation function from the experimentally measured surface topographies.

Figure 2 is an AFM image of an amorphous-Si film grown on a Si wafer using thermal evaporation. The detailed experiments have been described elsewhere.\(^{16} \) The image has a scale of \( 1 \times 1 \mu m^2 \) with data points of \( 512 \times 512 \). For this amorphous-Si surface, we have previously found that the roughness parameter \( \alpha \approx 1 \). The sampling height-height correlation functions are plotted as thin lines in Fig. 3(a). Different thin lines are calculated from different sampling im-

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**Amorphous-Si/Si(111) Surface**

![AFM image](image)

**FIG. 2.** AFM image (1 x 1 \( \mu m^2 \)) of amorphous Si film deposited at room temperature for \( t = 0.25 \) h using thermal evaporation technique. The deposition rate is \( \sim 1 \) Å/sec.
ages with each image being measured at a different place in the Si surface. The thick line represents the average of over ten such thin lines. The corresponding averaged power spectrum is also plotted in Fig. 3.

From Fig. 3, three important characteristics need to be emphasized.

(i) Each sampled $G_S(x)$ (corresponding to each thin-line plot) demonstrates oscillatory behavior. These oscillations are quite random.

(ii) The oscillation amplitude is significantly reduced after further averaging of many images, as shown in the thick line plot.

(iii) There is no split peak in the power spectrum.

A similar phenomenon also occurs in a different surface morphology measured by STM. Figure 4 is an STM image of a rough Si(111) surface after 500 eV Ar$^+$ sputtering for 60 min. The detailed report for this experiment will be published elsewhere.\textsuperscript{17} The image has a scale of 6300 $\times$ 6300 Å$^2$ with data points of 256 $\times$ 256. It has been determined that for this rough surface, the roughness parameter $\alpha$ is 0.7. Similarly, the sampling height-height correlation function $G_S(x)$ from the image shown in Fig. 4 is plotted in Fig. 5(a) as the thin line. The thick line is the average of over ten sampling $G_S(x)$ from different images. The averaged power spectrum is plotted in Fig. 5(b). One can easily find that the three characteristics mentioned in (i), (ii), and (iii) are also illustrated in Fig. 5.

From the above discussion, two questions need to be addressed.

(a) The random oscillation and fluctuation in each sampling $G_S(x)$ originated from the finite-size sampling data points may not represent the true statistical characteristics of the surface. The striking fact is that the number of terms to be averaged in these two cases is already very large ($N \sim 512 \times 512$ for the AFM data and $N \sim 256 \times 256$ for the STM data). If one used the strong law of large numbers to estimate the uncertainty for the sampling calculation, one could find that the relative uncertainty should be $\sim 0.2\%$ for the AFM case and $\sim 0.4\%$ for the STM case. However, the oscillation (or fluctuation) amplitudes shown in Figs. 3 and Fig. 5 are much higher than these estimates. As we have pointed out earlier, this discrepancy results from the correlation of the system, such as the correlation among the terms averaged in Eq. (4) and the correlation among the sampling data points measured from both AFM and STM. It is apparent that we need to establish a modified statistical law of large number to handle the correlated systems.

(b) From (ii) and (iii), we can conclude that both rough surfaces shown above are scale invariant. There should be no hidden cycles existing in these two systems. However, this conclusion would contradict the finding in (i). A possible explanation is that these apparent cycles exhibited from thin-line plots [Figs. 3(a) and 5(a)] might exist when the number of sampling data points is not sufficient for the average. These oscillations will diminish for sufficiently large number of the average, as shown in the thick-line plots. We shall show in the next section that the correct answers to these two questions can be given from an analytical rough surface model.

III. RANDOM GAUSSIAN SURFACE MODEL WITH $\alpha=1$

Consider a continuous, nondivergent, rough surface in which the surface height $z(r)$ is defined as
where the interface width can be calculated from Eq. (5), the integral notation \( \int_{-L/2}^{L/2} dr \) stands for \( \int_{-L/2}^{L/2} dx \) for \( d = 1 \), but for \( d = 2 \), \( \int_{-L/2}^{L/2} dr = \int_{-L/2}^{L/2} dx \int_{-L/2}^{L/2} dy \). In order to obtain analytical results, we can approximately replace the cutoff integral by the Gaussian integral,

\[
\frac{1}{L^d} \int_{-L/2}^{L/2} dr \to \frac{1}{L^d} \int_{-\infty}^{\infty} dr \exp \left[ -\frac{\pi r^2}{L^2} \right].
\]

Such approximation should be valid if the sampling size \( L \) is sufficiently large.

The sampling average height for the random Gaussian surface is thus calculated as

\[
h_s \approx \frac{1}{L^d} \int_{-\infty}^{\infty} dr \exp \left[ -\frac{\pi r^2}{L^2} \right] z(r).
\]

The mean square of the uncertainty for the sampling average height is then given by

\[
\langle [h_s - \langle h_s \rangle]^2 \rangle = w^2 \frac{1}{L^{2d}} \int_{-\infty}^{\infty} dr \int_{-\infty}^{\infty} dr' \exp \left[ -\frac{\pi r^2}{L^2} \right] \times \exp \left[ -\frac{\pi r'^2}{L^2} \right] \exp \left[ -\frac{(r-r')^2}{\xi^2} \right],
\]

where we have used the result from Eq. (7). Further calculation gives

\[
\langle [h_s - \langle h_s \rangle]^2 \rangle = \frac{\xi^d}{(\xi^2 + 2L^2/\pi)^{d/2}} w^2.
\]

In the case of \( L \gg \xi \), the standard deviation of the sampling average height is given by
\[ \Delta h_s \sim \sqrt{\left( [h_s - \langle h_s \rangle]_r^2 \right)^2} - w(\xi/L)^{d/2}. \]  

(11)

Equation (11) indicates that the sampling average height converges to its mean height within an order of \((\xi/L)^{d/2}\).

C. The convergence of the sampling interface width

The sampling interface width is given by

\[ w_S^2 = \frac{1}{L^2} \int_{-L/2}^{L/2} \int_{-L/2}^{L/2} \left( z(r) - h_S \right)^2 \exp \left[ -\frac{\pi r^2}{L^2} \right] dr d\mathbf{r} \]

\[ \times \exp \left[ -\frac{\pi r^2}{L^2} \right] [z(\mathbf{r})]^2 - h_S^2, \]  

(12)

where \( h_S \) is defined from Eq. (9b). For the random Gaussian model, we have

\[ \langle w_S^2 \rangle = w^2 - \frac{\xi^d}{(\xi^2 + 2L^2/\pi)^{d/2}} w^2. \]

Note that \( \langle w_S^2 \rangle \) is not exactly equal to \( w^2 \). This originates from the finite-size sampling effect. This effect leads to an offset of the average surface height by a quantity, \( \langle [h_S - \langle h_S \rangle]_r^2 \rangle = [\xi^d/(\xi^2 + 2L^2/\pi)^{d/2}] w^2 \).

The mean square of the uncertainty for the sampling interface width is given by

\[ \langle [w_S^2 - \langle w_S^2 \rangle]^2 \rangle = 2w^4 \frac{1}{L^2} \int_{-L/2}^{L/2} \int_{-L/2}^{L/2} \left( \exp \left[ -\frac{\pi r^2}{L^2} \right] \right) \left( \exp \left[ -\frac{\pi r^2}{L^2} \right] - \frac{(r-r')^2}{\xi^2} \right) + \langle h_S^2 \rangle - 2w^2 \langle h_S^2 \rangle - [\langle w_S^2 \rangle]^2. \]

Further calculation gives

\[ \langle [w_S^2 - \langle w_S^2 \rangle]^2 \rangle = 2w^4 \frac{\xi^d}{(\xi^2 + 2L^2/\pi)^{d/2}} \left[ \frac{\xi^d}{(\xi^2 + 2L^2/\pi)^{d/2}} + \frac{\xi^d}{(\xi^2 + 2L^2/\pi)^{d/2}} \right]. \]

In the case of \( L \gg \xi \), we have

\[ \Delta w_S^2 \sim \sqrt{\left( [w_s^2 - \langle w_s^2 \rangle]^2 \right)^2} \sim w^2 \frac{(\xi/L)^{d/2}}{2}. \]  

(13)

Again, it is indicated that the sampling interface width will converge to its mean within an order of \((\xi/L)^{d/2}\).

D. The convergence of the sampling height-height correlation function

For a continuous surface, the sampling height-height correlation function can be obtained by modifying Eq. (4) as

\[ G_s(\mathbf{r}) \approx \frac{1}{L^2} \int_{-L/2}^{L/2} ds \exp \left[ -\frac{\pi s^2}{L^2} \right] [z(\mathbf{r}+s) - z(\mathbf{s})]^2. \]

(14)

where we have assumed that \( L \gg r \), which is similar to the case of \( N_\times \gg m \).

Seemingly, \( \langle G_s(\mathbf{r}) \rangle = G(\mathbf{r}) \). Since we are more interested in how \( G_s(\mathbf{r}) \) converges to \( G(\mathbf{r}) \), we should consider the standard deviation of \( G_s(\mathbf{r}) \) given as

\[ \Delta^2 G(\mathbf{r}) = \langle [G_s(\mathbf{r}) - G(\mathbf{r})]^2 \rangle = \langle [G_s(\mathbf{r})]^2 \rangle - [G(\mathbf{r})]^2. \]

Equation (15) indicates that the sampling height-height correlation function \( G_s(\mathbf{r}) \) will converge to \( G(\mathbf{r}) \) within an order of \((\xi/L)^{d/2}\).

For the one-dimensional random Gaussian model surface, the sampling height-height correlation functions are plotted in Fig. 6. The thin dashed line represents \( G_s(\mathbf{r}) \) from an individual surface profile obtained from the Monte Carlo simulation. The thick solid line is an average of 10 such profiles (thin dashed line). The thick line is an average of 1000 profiles. We must emphasize that the kind of average is equivalent to the increase of the sampling size. For example, if \( M \) independent curves are averaged, the uncertainty will be reduced by a factor of \( 1/\sqrt{M} \) according to the strong law of large numbers. Since for each surface profile (corresponding to a thin dashed line), the uncertainty is \((\xi/L)^{1/2}\), the total uncertainty for the average is \((1/\sqrt{M}) \sqrt{\xi/L} = \sqrt{\xi/(ML)}\). This indicates that the average of \( M \) independent curves is equivalent to enlarging a single original sampling size \( L \) by a factor of \( M \), i.e., \( ML \).

Figure 6 clearly demonstrates how the sampling correlation function \( G_s(\mathbf{r}) \) converges to the true height-height correlation function \( G(\mathbf{r}) \) when the sampling size increases. This is also consistent with our experimental data shown in Figs. 3 and 5.
The normalized curves, $H(u,r)/G(u+r)G(r)$ against $u$, for $r=0.75\xi$, $\xi$, $1.5\xi$, $2\xi$, and $4\xi$, respectively, where $H(u,r)$ is the autocorrelation for the sampling function $G_S(r)$, defined in Eq. (16).

E. Sampling-induced apparent cycle in height-height correlation function

In Fig. 6, the sampling height-height correlation function also demonstrates the oscillatory behavior at $r>\xi$. These oscillations will diminish when $(\xi/L)^{d/2}\rightarrow 0$, as shown both in Sec. III D and Fig. 6. To find out whether or not there exist hidden cycles in these random oscillations, we can calculate the autocorrelation for the sampling function $G_S(r)$, which is defined as

$$H(u,r) = \langle [G_S(u+r) - G(u+r)][G_S(r) - G(r)] \rangle$$

$$= \langle G_S(u+r)G_S(r) \rangle - G(u+r)G(r).$$

The detailed calculation of $H(u,r)$ for the random Gaussian model is given by

$$H(u,r) \approx 2^{-d} \pi^{d/2} \xi^d \left[ 2 - 2 \exp \left( -\frac{r^2}{2\xi^2} \right) \right]$$

$$+ \exp \left[ -2 \frac{(r+u)^2}{\xi^2} \right] + \exp \left[ -\frac{u^2}{2\xi^2} \right]$$

$$- 2 \exp \left[ -2 \frac{(r+u)^2}{\xi^2} \right],$$

for $L\gg r$ and $L\gg u>0$. (17)

To show the behavior of the autocorrelation function $H(u,r)$, we plot in Fig. 7 the normalized curves, $H(u,r)/G(u+r)G(r)$ against $u$, for $r=0.75\xi$, $\xi$, $1.5\xi$, $2\xi$, and $4\xi$, respectively. From Eq. (17) and Fig. 7, several interesting points can be addressed.

(i) For $u>\xi$, $H(u,r)$ decays to a constant which depends on $r$,

$$H(u,r) \mid_{u>\xi} = 2^{-d} \pi^{d/2} \xi^d \left[ 1 - \exp \left( -\frac{r^2}{2\xi^2} \right) \right].$$

The nonzero constant means that the fluctuation in the sampling correlation function is always self-correlated no matter how large $u$ can be. This is not surprising, because the sampling calculation of $G_S(r)$ involves every data point in the sampling image, as indicated in Eqs. (4) and (14). This is the main reason that $G_S(r)$ has long-range correlation.

(ii) Figure 7 shows that in the vicinity of $r=\xi$, such as at $r=0.75\xi$, $\xi$, and $1.5\xi$, the $H(u,r)$ curves exhibit a profound shape, where $H(u,r)$ increases with $u$ at $0\leq u<\xi$ and then decreases until they reach constants at large $u$. This suggests that the oscillation and fluctuation of the sampling correlation $G_S(r)$ might have a regular cycle with a characteristic length scale $\xi$ in the vicinity of the turning point, $r=\xi$. This hidden cycle can be seen clearly nearby the turning point both in the modeling curve in Fig. 6 and the experimental curves shown in Figs. 3 and 5. Since the oscillation amplitude diminishes as $(\xi/L)^{d/2}\rightarrow 0$, this hidden cycle must result from the sampling process in the correlated surface. We might characterize this cycle as a sampling induced cycle.

(iii) Away from the turning point, the plot of $H(u,r)$ versus $u$ only exhibits a monotonic decay to a constant, as shown in Fig. 7 at $r=4\xi$. It does not show a bump as it does at $r=0.75\xi$, $\xi$, and $1.5\xi$. This indicates that for $r>\xi$, the oscillations in $G_S(r)$ have a much more random feature and cannot be characterized by a single length scale. This can be directly examined from Eq. (17), where for $r>4\xi$, one has

$$H(u,r) \mid_{r>\xi} \approx 2^{-d} \pi^{d/2} \xi^d \left[ 2 + \exp \left( -\frac{u^2}{2\xi^2} \right) \right].$$

It is shown that $H(u,r)$ at $r>\xi$ will monotonically decay with $u$ to a constant within a range of $\xi$. Therefore, for $r>\xi$, what we can say is that the random oscillations in $G_S(r)$ have periodic length scales which range from 0 to $\sim \xi$.

IV. DISCUSSION

Our proposed random Gaussian model has provided analytical solutions for the statistical sampling process in correlated systems. This model can also give answers to the questions appearing in our experimental analysis in the AFM and STM studies.

The random Gaussian model indicates that the sampling average surface height, the sampling interface width, and the sampling height-height correlation function do not converge to their true values following the rule of $1/\sqrt{N}$ as in the case of independent systems. Instead, as shown in Eqs. (11), (13), and (15), respectively, they approach their means within an order of $(\xi/L)^{d/2}$. In this model, the system has sampling data points, $N=L^d$. Remember that both $\xi$ and $L$ are in units of the spacing between the neighboring data points. $L$ is also the number of data points along a corresponding direction. The above conclusion is thus consistent with the law of large numbers for the correlated systems: the average of $N$-correlated variables having a correlation length $\xi$ will converge to their mean within an order of $\sqrt{\xi}/N$.

A very important practical issue arises from the above conclusion, which is the accuracy of statistical averages in a correlated system. The accuracy might not only depend on how many data points one has, but more depends on how big the sampling size can be. In other words, it is the ratio
\( \xi/L \), not the number of the data points, that determines the accuracy. Once the ratio \( \xi/L \) is settled, one may not be able to increase the accuracy no matter how many data points one can collect. This rule is distinctly different from the strong law of large numbers \( 1/\sqrt{N} \) for independent random variables. One must realize this important difference when dealing with correlated systems.

Another important practical issue is how to distinguish the self-affine surface from the mound surface. The existence of sampling-induced oscillation in the height-height correlation function suggests that one should be cautious when a mound surface is claimed. A statistical average of a large number of images is required, and the condition \( \sqrt{\xi/L} \ll 1 \) for sampling must also be imposed. To illustrate this idea, here we present a numerical example. Two sampling surface images shown in Fig. 8 were calculated from a similar surface model (see the Appendix), one is a mound surface, and the other is a self-affine surface. In order to compare these two, the lateral lengths chosen are about the same (~20 arbitrary units). A visual inspection of Fig. 8 reveals that a mound surface has regular mounds with the same size, while a self-affine surface has mounds of various sizes. However, judging from the height-height correlation functions in Fig. 9, one can hardly tell any difference between these two curves. Both height-height correlation functions have oscillations with almost the same cycling length. If one does not have a prior knowledge about these two surfaces, one may claim that they might be the same. Hence, Figs. 8 and 9 demonstrate again that an examination of only one image and its height-height correlation function is not enough to distinguish between different surface morphologies. A statistics of a large number of images is needed. The thick lines in Figs. 10(a) and 10(b) are the height-height correlation functions averaged from ten images for the mound surfaces and ten images for the self-affine surfaces, respectively. All the thin lines represent height-height correlation functions calculated from indi-

FIG. 8. The simulated sampling images (in arbitrary scales) of (a) a self-affine surface and (b) a mound surface. A detailed numerical algorithm is explained in the Appendix.

FIG. 9. The height-height correlation functions of sampling images for (a) self-affine surface and (b) a mound surface. Note that both curves have a similar oscillatory behavior.

FIG. 10. The height-height correlation functions for (a) ten self-affine surfaces and (b) ten mound surfaces. A thin line represents the height-height correlation function for one individual image, and the thick line is the average of ten images.
vidual images. The average height-height correlation function for a mound surface shows a profound oscillation, while the oscillation for a self-affine surface diminishes compared to individual ones. Another feature is that the oscillation for a self-affine surface is very random, but is regular for a mound surface, especially for the first cycle which is in phase. Therefore, in order to distinguish a mound surface from a self-affine surface unambiguously, both large number statistics of images and the sampling condition $\sqrt{\xi^2/N} \ll 1$ are required.

We have shown both experimentally and theoretically that random oscillations do exist in the height-height correlation function due to an undersampling practice. In the proposed model, we indicate that sampling-induced periodic cycles are real in a scale-invariant rough surface. The oscillation amplitude approaches zero in an order of $(\xi/L)^{d/2}$. The existence of sampling-induced hidden cycles has a profound impact on the statistical data analysis process. It might cause problems in the search of periodic cycles hidden by noises or irregular perturbations. For a correlated system, these hidden cycles might originate from the undersampling practice, and thus might be artificial. For example, in the case of correlated rough surfaces discussed above, one might mistakenly consider these sampling-induced cycles to be the evidence of the regular mound structures. This reminds us to be cautious when integrating and interpreting the experimental data. In order to distinguish a mound surface from a self-affine surface, the sampling condition $\sqrt{\xi^2/N} \ll 1$ and an average of a large number of images are required.

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APPENDIX

Based on the linear Langevin equation, $\partial^2 z/\partial t^2 = v \nabla^2 z - \kappa \nabla^2 z + \eta(r, t)$, both the mound surface and the self-affine surface can be simulated from the following linear system:

$$\kappa \nabla^4 z - v \nabla^2 z + \gamma z = \eta(r),$$  \hspace{1cm} (A1)

where $\kappa$, $v$, and $\gamma$ are coefficients, and $\eta(r)$ is a Gaussian white noise, satisfying

$$\langle \eta(r) \rangle = 0,$$

$$\langle \eta(r) \eta(r') \rangle = \delta(r - r').$$  \hspace{1cm} (A2)

The third term on the left-hand side of Eq. (A1) originates from the time-dependent term in the Langevin equation, and partially determines how random the simulated surface is. The solution for Eq. (A1) is

$$z(r) = \int \frac{\Theta(q)}{\kappa q^4 + v q^2 + \gamma} e^{i q \cdot r} d q,$$

where $\Theta(q) = \int \eta(r) e^{-i q \cdot r} d r$. The corresponding power spectrum is

$$P(q) = \frac{1}{(\kappa q^4 + v q^2 + \gamma)^2} = \frac{1}{T^2 + q^2 + \text{sgn}(\frac{v}{\kappa}) q^2},$$  \hspace{1cm} (A4)

where $T^2 = \gamma - v^2/4 \kappa$ and $q_0 = (\frac{1}{2} |v/\kappa|)^{1/2}$. If $v/\kappa < 0$, Eq. (A1) generates a mound surface with an average mound separation $2 \pi/q_0$, and $T^2$ determines how well the mounds are separated. The larger the $T^2$ value, the more random the mounds distribution. If $v/\kappa > 0$, then Eq. (A1) gives a self-affine surface, with a roughness exponent $\alpha \sim 1$ and a lateral correlation length $\xi$ determined by both $T^2$ and $q_0$.

The numerical algorithm is the following: first, generate a random phase noise $\Theta(q) = e^{i \theta}$, where $\theta$ is a uniform random noise between 0 and $2 \pi$, and then from Eq. (A3), the amplitude of a simple fast Fourier transform of $\Theta(q)/(\kappa q^4 + v q^2 + \gamma)$ will give the desired surface morphology.

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1 Present address: Sharp Microelectronics Technology, INC, 5700 NW Pacific Rim Blvd., Camas, WA 98607.

2 Present address: Applied Materials, 2450 Walsh Ave., M/S 8037, Santa Clara, CA 95051.


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One may prefer to use the autocorrelation function $R(r)$ or the power spectrum $P(q)$ of the surface instead of the height-height correlation function $G(r)$. However, these three functions are essentially equivalent to each other for describing a rough surface: $G(r) = 2w^2 - 2R(r)$, and $P(q) = \int R(r)e^{-iqr}dr$. For $R(r)$, the distinction between a mound surface and a self-affine surface is the oscillatory behavior and the existence of a zero-crossing point. For $P(q)$, the distinction is the existence of a splitting peak in the spectrum for the mound surface.