6-36.
c chart:
\[ CL = \bar{c} = 2.36 \]
\[ UCL = \bar{c} + 3\sqrt{\bar{c}} = 2.36 + 3\sqrt{2.36} = 6.97 \]
\[ LCL = \bar{c} - 3\sqrt{\bar{c}} = 2.36 - 3\sqrt{2.36} \approx 0 \]

**MTB : Stat : Control Charts : C Chart : Ex6-36Num**

C Chart for Surface Defects

![Chart](image)

TEST 1. One point more than 3.00 sigmas from center line.
Test Failed at points: 13

No. The plate process does not seem to be in statistical control.

6-37.

\[ CL = \bar{u} = 0.7007 \]
\[ UCL_i = \bar{u} + 3\sqrt{\bar{u}/n_i} = 0.7007 + 3\sqrt{0.7007/n_i} \]
\[ LCL_i = \bar{u} - 3\sqrt{\bar{u}/n_i} = 0.7007 - 3\sqrt{0.7007/n_i} \]

<table>
<thead>
<tr>
<th>( n_i )</th>
<th>([LCL_i, UCL_i])</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>[0.1088, 1.2926]</td>
</tr>
<tr>
<td>20</td>
<td>[0.1392, 1.2622]</td>
</tr>
<tr>
<td>21</td>
<td>[0.1527, 1.2487]</td>
</tr>
<tr>
<td>22</td>
<td>[0.1653, 1.2361]</td>
</tr>
<tr>
<td>24</td>
<td>[0.1881, 1.2133]</td>
</tr>
</tbody>
</table>
The process appears to be in statistical control. I would recommend using a standardized control chart for controlling current production in cases.

6-38.

\[
\begin{align*}
CL &= \bar{u} = 0.7007; \quad \bar{n} = 20.55 \\
UCL &= \bar{u} + 3\sqrt{\frac{\bar{u}}{\bar{n}}} = 0.7007 + 3\sqrt{\frac{0.7007}{20.55}} = 1.2547 \\
LCL &= \bar{u} - 3\sqrt{\frac{\bar{u}}{\bar{n}}} = 0.7007 - 3\sqrt{\frac{0.7007}{20.55}} = 0.1468
\end{align*}
\]

U Chart for Paper Imperfections, avg \( n = 20 \)
6-39.

\[ z_i = \frac{(u_i - \bar{u})}{\sqrt{\frac{s}{n_i}}} = \frac{(u_i - 0.7007)}{\sqrt{0.7007 / n_i}} \]

MTB : Stat : Control Charts : I Chart : Ex6-39zi

Standardized Chart for Paper Imperfections

6-40.

c chart based on # of nonconformities per cassette deck

\[ CL = \bar{c} = 1.5 \]

\[ UCL = \bar{c} + 3\sqrt{\frac{\bar{c}}{n}} = 1.5 + 3\sqrt{1.5} = 5.17 \]

\[ LCL \approx 0 \]

MTB : Stat : Control Charts : C Chart : Ex6-40Num

C Chart for Cassette Decks

Process is in statistical control. Use these limits to control future production.
6-42.
(a) The new inspection unit is \( n = 4 \) cassette decks. A \( c \) chart of the total number of nonconformities per inspection unit is appropriate.

\[
\text{CL} = n\bar{c} = 4(1.5) = 6
\]

\[
\text{UCL} = n\bar{c} + 3\sqrt{n\bar{c}} = 6 + 3\sqrt{6} = 13.35
\]

\[
\text{LCL} = n\bar{c} - 3\sqrt{n\bar{c}} = 6 - 3\sqrt{6} \approx 0
\]

(b) The sample is \( n = 1 \) new inspection units. A \( u \) chart of average nonconformities per inspection unit is appropriate.

\[
\text{CL} = \bar{u} = \frac{\text{total nonconformities}}{\text{total inspection units}} = \frac{27}{(18/4)} = 6.00
\]

\[
\text{UCL} = \bar{u} + 3\sqrt{\bar{u}/n} = 6 + 3\sqrt{6/1} = 13.35
\]

\[
\text{LCL} = \bar{u} - 3\sqrt{\bar{u}/n} = 6 - 3\sqrt{6/1} \approx 0
\]

6-43.
(a) The new inspection unit is \( n = 2500/1000 = 2.5 \) of the old unit. A \( c \) chart of the total number of nonconformities per inspection unit is appropriate.

\[
\text{CL} = n\bar{c} = 2.5(6.17) = 15.43
\]

\[
\text{UCL} = n\bar{c} + 3\sqrt{n\bar{c}} = 15.43 + 3\sqrt{15.43} = 27.21
\]

\[
\text{LCL} = n\bar{c} - 3\sqrt{n\bar{c}} = 15.43 - 3\sqrt{15.43} = 3.65
\]

The plot point, \( \hat{c} \), is the total number of nonconformities found while inspecting a sample 2500m in length.

(b) The sample is \( n = 1 \) new inspection units. A \( u \) chart of average nonconformities per inspection unit is appropriate.

\[
\text{CL} = \bar{u} = \frac{\text{total nonconformities}}{\text{total inspection units}} = \frac{111}{(18 \times 1000)/2500} = 15.42
\]

\[
\text{UCL} = \bar{u} + 3\sqrt{\bar{u}/n} = 15.42 + 3\sqrt{15.42/1} = 27.20
\]

\[
\text{LCL} = \bar{u} - 3\sqrt{\bar{u}/n} = 15.42 - 3\sqrt{15.42/1} = 3.64
\]

The plot point, \( \hat{u} \), is the average number of nonconformities found in 2500m, and since \( n = 1 \), this is the same as the total number of nonconformities.
6-44.  
(a)  
A c chart of total nonconformities with a sample of \( n = 1 \) inspection units is appropriate.  
\[ \text{CL} = \bar{c} = \sum D / m = 27 / 16 = 1.6875 \]  
\[ \text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 1.6875 + 3\sqrt{1.6875} = 5.585 \]  
\[ \text{LCL} \Rightarrow 0 \]  

MTB : Stat : Control Charts : C Chart : Ex6-44Num  

![C Chart for Manual Transmissions](image)

(b)  
The process is in statistical control.

(c)  
The new sample is \( n = 8/4 = 2 \) inspection units.  
\[ \text{CL} = n\bar{c} = 2(1.6875) = 3.375 \]  
\[ \text{UCL} = n\bar{c} + 3\sqrt{n\bar{c}} = 3.375 + 3\sqrt{3.375} = 8.886 \]  
\[ \text{LCL} \Rightarrow 0 \]  

6-45.  
(a)  
\[ \text{CL} = \bar{c} = 4 \]  
\[ \text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 4 + 3\sqrt{4} = 10 \]  
\[ \text{LCL} = \bar{c} - 3\sqrt{\bar{c}} = 4 - 3\sqrt{4} \Rightarrow 0 \]  

(b)  
\( c = 4; \) \( n = 4 \)  
\[ \text{CL} = \bar{u} = c / n = 4 / 4 = 1 \]  
\[ \text{UCL} = \bar{u} + 3\sqrt{\bar{u}/n} = 1 + 3\sqrt{1/4} = 2.5 \]  
\[ \text{LCL} = \bar{u} - 3\sqrt{\bar{u}/n} = 1 - 3\sqrt{1/4} \Rightarrow 0 \]
6-46.
Use the cumulative Poisson tables.
\( \bar{x} = 16 \)

\[ \Pr\{ x \leq 21 \mid c = 16 \} = 0.885; \text{UCL} = 21 \]
\[ \Pr\{ x \leq 10 \mid c = 16 \} = 0.096; \text{LCL} = 10 \]

6-47.
(a)
CL = \( \bar{c} = 9 \)

UCL = \( \bar{c} + 3\sqrt{\bar{c}} = 9 + 3\sqrt{9} = 18 \)

LCL = \( \bar{c} - 3\sqrt{\bar{c}} = 9 - 3\sqrt{9} = 0 \)

(b)
\( c = 16; \quad n = 4 \)

CL = \( \bar{u} = c/n = 16/4 = 4 \)

UCL = \( \bar{u} + 3\sqrt{\bar{u}/n} = 4 + 3\sqrt{4}/4 = 7 \)

LCL = \( \bar{u} - 3\sqrt{\bar{u}/n} = 4 - 3\sqrt{4}/4 = 1 \)

6-48.
\( u \) chart with \( u = 6.0 \) and \( n = 3 \). \( c = u \times n = 18 \).
Find limits such that \( \Pr\{ D \leq \text{UCL} \} = 0.980 \) and \( \Pr\{ D < \text{LCL} \} = 0.020 \). From the cumulative Poisson tables

<table>
<thead>
<tr>
<th>( x )</th>
<th>( \Pr{ D \leq x \mid c = 18 } )</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>0.015</td>
</tr>
<tr>
<td>10</td>
<td>0.030</td>
</tr>
<tr>
<td>26</td>
<td>0.972</td>
</tr>
<tr>
<td>27</td>
<td>0.983</td>
</tr>
</tbody>
</table>

UCL = \( x/n = 27/3 = 9 \), and LCL = \( x/n = 9/3 = 3 \). As a comparison, the normal distribution gives

UCL = \( \bar{u} + z_{0.025} \sqrt{\bar{u}/n} = 6 + 2.054 \sqrt{6/3} = 8.905 \)

LCL = \( \bar{u} - z_{0.025} \sqrt{\bar{u}/n} = 6 - 2.054 \sqrt{6/3} = 3.095 \)

6-49.
Using the cumulative Poisson distribution

<table>
<thead>
<tr>
<th>( x )</th>
<th>( \Pr{ D \leq x \mid c = 7.6 } )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.019</td>
</tr>
<tr>
<td>3</td>
<td>0.055</td>
</tr>
<tr>
<td>12</td>
<td>0.954</td>
</tr>
<tr>
<td>13</td>
<td>0.976</td>
</tr>
</tbody>
</table>

for the \( c \) chart, UCL = 13 and LCL = 2. As a comparison, the normal distribution gives

UCL = \( \bar{c} + z_{0.025} \sqrt{\bar{c}} = 7.6 + 1.96\sqrt{1.6} = 13.00 \)

LCL = \( \bar{c} - z_{0.025} \sqrt{\bar{c}} = 7.6 - 1.96\sqrt{1.6} = 2.20 \)
6-53.
A c chart with one inspection unit equal to 50 manufacturing units is appropriate.
\( \bar{c} = \frac{850}{100} = 8.5 \). From the cumulative Poisson distribution,
\[
\begin{array}{c|c}
 x & \Pr\{D \leq x \mid c = 8.5\} \\
3 & 0.030 \\
13 & 0.949 \\
14 & 0.973 \\
\end{array}
\]

LCL = 3 and UCL = 13.

For comparison, the normal distribution gives

\[
\begin{align*}
\text{UCL} &= \bar{c} + z_{0.01} \sqrt{\bar{c}} = 8.5 + 1.88 \sqrt{8.5} = 13.98 \\
\text{LCL} &= \bar{c} + z_{0.01} \sqrt{\bar{c}} = 8.5 - 1.88 \sqrt{8.5} = 3.02 \\
\end{align*}
\]

6-54.
(a) Plot the number of nonconformities per water heater on a c chart.

\[
\begin{align*}
\text{CL} &= \bar{c} = \frac{\sum D}{m} = \frac{924}{176} = 5.25 \\
\text{UCL} &= \bar{c} + 3\sqrt{\bar{c}} = 5.25 + 3\sqrt{5.25} = 8.25 \\
\text{LCL} &= 0 \\
\end{align*}
\]

Plot the results after inspection of each water heater, approximately 8/day.

(b) Let new inspection unit \( n = 2 \) water heaters

\[
\begin{align*}
\text{CL} &= n\bar{c} = 2(5.25) = 10.5 \\
\text{UCL} &= n\bar{c} + 3\sqrt{n\bar{c}} = 10.5 + 3\sqrt{10.5} = 20.22 \\
\text{LCL} &= n\bar{c} - 3\sqrt{n\bar{c}} = 10.5 - 3\sqrt{10.5} = 0.78 \\
\end{align*}
\]

(c)

\[
\begin{align*}
\Pr\{\text{type I error}\} &= \Pr\{D < \text{LCL} \mid c\} + \Pr\{D > \text{UCL} \mid c\} \\
&= \Pr\{D < 0.78 \mid 10.5\} + 1 - \Pr\{D \leq 20.22 \mid 10.5\} \\
&= \text{POI}(0,10.5) + 1 - \text{POI}(20,10.5) = 0.000 + 1 - 0.990 = 0.010 \\
\end{align*}
\]

6-55.
\( \bar{c} = 4.0 \) average number of nonconformities/unit. Desire \( \alpha = 0.99 \). Use the cumulative Poisson distribution to determine the UCL.

\[
\begin{array}{c|c} 
 x & \Pr\{\text{CL} \leq x\} \\
0 & 0.018 \\
1 & 0.092 \\
2 & 0.238 \\
3 & 0.433 \\
4 & 0.629 \\
5 & 0.785 \\
\end{array}
\quad
\begin{array}{c|c} 
 x & \Pr\{\text{CL} \leq x\} \\
6 & 0.889 \\
7 & 0.949 \\
8 & 0.979 \\
9 & 0.992 \\
10 & 0.997 \\
11 & 0.999 \\
\end{array}
\]

An UCL = 9 will give a probability of 0.992 of concluding the process is in control, when in fact it is.
6-56.
Use a $c$ chart for nonconformities with an inspection unit $n = 1$ refrigerator. 

$\sum D = 16$ in 30 refrigerators; $\bar{c} = 16 / 30 = 0.533$

(a) 
3-sigma limits are $\bar{c} \pm 3\sqrt{\bar{c}} = 0.533 \pm 3\sqrt{0.533} = [0, 2.724]$

(b) 
$\alpha = \Pr\{D < \text{LCL} | c\} + \Pr\{D > \text{UCL} | c\} = \Pr \{D < 0 | 0.533\} + 1 - \Pr \{D \leq 2.72 | 0.533\} 
= 0 + 1 - \text{POI}(2, 0.533) = 1 - 0.983 = 0.017$

where $\text{POI}(\cdot)$ is the cumulative Poisson distribution.

(c) 
$\beta = \Pr\{\text{not detecting shift}\} = \Pr \{D < \text{UCL} | c\} - \Pr \{D \leq \text{LCL} | c\}$

$\Pr \{D < 2.72 | 2.0\} - \Pr \{D \leq 0 | 2.0\} = \text{POI}(2, 2) - \text{POI}(0, 2)$

$= 0.6767 - 0.1353 = 0.5414$

where $\text{POI}(\cdot)$ is the cumulative Poisson distribution.

(d) 
$\text{ARL}_1 = \frac{1}{1 - \beta} = \frac{1}{1 - 0.541} = 1.847 \approx 2$

6-57.
$\bar{c} = 0.533$

(a) 
$\bar{c} \pm 2\sqrt{\bar{c}} = 0.533 \pm 2\sqrt{0.533} = [0, 1.993]$

(b) 
$\alpha = \Pr\{D < \text{LCL} | \bar{c}\} + \Pr\{D > \text{UCL} | \bar{c}\} = \Pr \{D < 0 | 0.533\} + 1 - \Pr \{D \leq 1.993 | 0.533\}$

$= 0 + 1 - \text{POI}(1, 0.533) = 1 - 0.8996 = 0.1004$

where $\text{POI}(\cdot)$ is the cumulative Poisson distribution.

(c) 
$\beta = \Pr\{D < \text{UCL} | \bar{c}\} - \Pr\{D \leq \text{LCL} | \bar{c}\} = \Pr \{D < 1.993 | 2\} - \Pr \{D \leq 0 | 2\}$

$= \text{POI}(1, 2) - \text{POI}(0, 2) = 0.406 - 0.135 = 0.271$

where $\text{POI}(\cdot)$ is the cumulative Poisson distribution.

(d) 
$\text{ARL}_1 = \frac{1}{1 - \beta} = \frac{1}{1 - 0.271} = 1.372 \approx 2$