Solutions to Problems 1, 2 and 7 followed by 3, 4, 5, 6 and 8.

1. The cause-and-effect diagram below was created by a department of the 3M Company to analyze the variability in the adhesion of a particular product,

We have added “concentration” as a branch off “adhesion,” and “mixing method” as a branch off “concentration.” Any number of causes could be added to existing branches and sub-branches.
2. Following are MINITAB-produced Pareto diagrams for each week and a Pareto diagram for the three-week total. Discussion follows the graphs.

Pareto Diagram for Week 1

Pareto Diagram for Week 2

Pareto Diagram for Week 3
The primary causes in order of frequency of occurrence, and hence presumably in importance are “sand,” “corebreak,” and “broken.” The reason for plotting both the individual weekly totals and the entire three-week total is to see whether there is consistency over time with respect to order of frequency. Here, the order of the three most frequent causes is the same for each week as well as the entire period, and the percentages are remarkably consistent from week to week as well as for the period. If the causes and/or their frequencies showed high variability, this would reveal something different about the process. For example, suppose different types of castings were produced from week to week and the primary causes were also changing. This would suggest that different causes are associated with different casting types.
7. The $X$-bar and $S$ charts appear below. The process appears to be in control (as before for the $X$-bar and $R$ charts). Note, however, that on the $X$-bar the limits are slightly different, due of course to the different method of estimating the process standard deviation. Here, the estimate of the standard deviation is

$$\hat{\sigma} = \bar{s}/c_4 = 2.703/0.9213 = 2.93,$$

slightly less than the $r$-based estimate.

Calculation of the limits for a 0.998 probability limit $S$-chart follows. From eqn. (31) and Table 3.1, Notes,

$$[LCL_s, UCL_s] \equiv \left[ \frac{\bar{s}}{c_4} \sqrt{\frac{\chi^2_{0.998}}{n-1}}, \frac{\bar{s}}{c_4} \sqrt{\frac{\chi^2_{0.001}}{n-1}} \right] \equiv \left[ 2.93 \sqrt{\frac{0.024}{3}}, 2.93 \sqrt{\frac{16.266}{3}} \right] \equiv [0.262, 6.822].$$

Compare this with the Shewhart limits above. Note that although small, there is a positive lower limit, and the upper limit is greater than the Shewhart upper limit. Here, these differences are small, and there is no difference with respect to out-of-control indications. In some cases, however, the difference in the limits may be just enough to cause a difference with respect to points being within the limits on one type chart while being outside of the limits on the other.
4-23. Check:
Any point outside the 3-sigma control limits? NO. (Point #12 is within the lower 3-sigma control limit.)
2 of 3 beyond 2 sigma of centerline? YES, points #16, 17, and 18.
4 of 5 at 1 sigma or beyond of centerline? YES, points #5, 6, 7, 8, and 9.
8 consecutive points on one side of centerline? NO.
Two out-of-control criteria are satisfied.

4-24. The pattern in Figure (a) matches the control chart in Figure (2).
The pattern in Figure (b) matches the control chart in Figure (4).
The pattern in Figure (c) matches the control chart in Figure (5).
The pattern in Figure (d) matches the control chart in Figure (1).
The pattern in Figure (e) matches the control chart in Figure (3).

4-25. From Figure 4-3, UCL = 74.0135, LCL = 73.9865, and $\bar{x} = 74.0000$.

(a) $\sigma_x = \frac{UCL - \bar{x}}{3} = \frac{74.0135 - 74.0000}{3} = 0.0045$
$UCL_{2-S} = \bar{x} + 2\sigma_x = 74.0000 + 2(0.0045) = 74.0090$
$LCL_{2-S} = \bar{x} - 2\sigma_x = 74.0000 - 2(0.0045) = 73.9910$

(b) Visual examination of the $\bar{x}$ chart in Figure 4-3 shows no point exceeds the 3-sigma control limits, but three points (#6, 8, and 14) exceed the 2-sigma warning limits. This is an increase in the number of false alarms.

(c) $ARL_{0/2-S} = 1/0.046 \approx 2$
$ARL_{0/3-S} = 1/0.0027 \approx 370$
Using 2-sigma limits would shorten the in-control ARL.

(d) Figure 4-3 illustrates the effect of using narrower limits than the 3-sigma control limits - the number of false alarms will increase, the in-control ARL will shorten, and there will be more investigations for assignable causes. Whether this is acceptable depends on the penalty cost for non-conforming product. Are the costs of extra investigations less than the costs of increased nonconformities?

4-26. Rule 1:
$\alpha_1 = Pr\{\text{out-of-control}\} = Pr\{1 \text{ of 7 beyond}\} + Pr\{2 \text{ of 7 beyond}\} + \cdots + Pr\{7 \text{ of 7 beyond}\}$
$= 1 - Pr\{0 \text{ of 7 beyond}\} = 1 - \binom{7}{0}(0.0027)^0(1 - 0.0027)^7 = 1 - 0.98125 = 0.01875$

Rule 2:
$\alpha_2 = Pr\{\text{out-of-control}\} = Pr\{\text{all 7 on one side}\} = (0.5)^7 = 0.00781$
Chapter 5 Exercise Solutions

MINITAB™ Release 13.1 Worksheet “CHAP05.MTW”
Microsoft® Excel 2000 Workbook “CHAP05.XLS”

Note: MINITAB defines sensitizing rules for control charts slightly differently than the standard rules. In particular, a run of \( n \) consecutive points on one side of the center line defaults to 9, instead of 8. This can be changed by selecting: Stat : Control Charts : Define Tests : Test 2, then changing the argument \( K \) to 8. This only lasts for the current session, and rules are reset to default at exit.

5-1.
XLS : CHAP05.XLS : worksheet Ex 5-1
(a) \( n = 5; \ \bar{x} = 34.00; \ \bar{R} = 4.71 \)

The process is not in statistical control; \( \bar{x} \) is beyond the upper control limit for both Sample No. 12 and Sample No. 15. With these two samples excluded from the control limit calculations:
\[
\bar{x} = 33.65; \ \bar{R} = 4.5; \ \hat{\sigma} = \bar{R} / d_2 = 4.5 / 2.326 = 1.93
\]
The process is in statistical control with no out-of-control signals, runs, trends, or cycles.
5-2 continued
(b) 
\[ n = 4, \bar{X} = 10.38, \bar{R} = 6.25, \hat{\sigma} = \bar{R} / d_2 = 6.25 / 2.059 = 3.04 \]
Actual specs are 350 V ± 5 V
\[ P_{CR} = \frac{USL - LSL}{6\hat{\sigma}} = \frac{355 - 345}{6(3.04 / 10)} = 5.48 \text{, so the process is capable.} \]

Also:
**MTB : Stat : Quality Tools : CAPA for Ex5-2V**
Using transformation \( x_i = (\text{observed voltage on unit } i - 350) \times 10 \), USLT = +50, LSLT = −50

(c)  
**MTB : Graph : Prob Plot for Ex5-2V**

A normal probability plot of the transformed output voltage shows the distribution is close to normal.
5-4.
(a)

MTB : Stat : Control Charts : Xbar/R Chart: Ex5-4Th

Xbar/R Chart for Board Thickness

Test Results for Xbar Chart
TEST 1. One point more than 3.00 sigmas from center line.
Test Failed at points: 22

Test Results for R Chart
TEST 1. One point more than 3.00 sigmas from center line.
Test Failed at points: 15

Assuming an assignable cause is found, remove samples 15 and 22:

Revised Xbar Chart for Board Thickness
5-4 (a) continued

Revised R Chart for Board Thickness

\[ \bar{R} = 8.48 \times 10^{-4} \]

\[ UCL = 0.002183 \]

\[ LCL = 0 \]

(b) 
\[ \hat{\sigma}_s = \frac{\bar{R}}{d_2} = \frac{0.00092}{1.693} = 0.00054 \]

(c) 
Natural tolerance limits are: 
\[ \bar{x} \pm 3\hat{\sigma}_s = 0.06295 \pm 3(0.00054) = [0.06133, 0.06457] \]

(d) 
\[ C_p = \frac{USL - LSL}{6\hat{\sigma}_s} = \frac{+0.0015 - (-0.0015)}{6(0.00054)} = 0.93 \]

Also, 

MTB : Stat : Quality Tools : CAPA for Ex5-4Th

Process Capability Analysis for Board Thickness

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<tr>
<th>Process Data</th>
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<th>Within</th>
<th>Overall</th>
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<tr>
<td>Target</td>
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<tr>
<td>LSL</td>
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<td>Mean</td>
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<tr>
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<tr>
<td>StDev (Overall)</td>
<td>0.0006220</td>
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<td></td>
</tr>
</tbody>
</table>

Potential (Within) Capability:
- Cp: 0.92
- CPL: 0.95
- Cpk: 0.89
- CPL: 0.83

Cp: 0.92
CPL: 0.95
Cpk: 0.89
5-10 continued

(c) 
\[ \hat{PCR} = \frac{USL - LSL}{6\sigma_x} = \frac{+5.0 - (-5.0)}{6(1.756)} = 0.95 \text{, so the process is not capable.} \]

(d) 
\[ \hat{p}_{\text{scrap}} = Pr\{x < LSL\} = Pr\{x < 36\} = \Phi \left( \frac{36 - 40}{1.756} \right) = \Phi(-2.28) = 0.01137 \text{, or 1.3\%.} \]
\[ \hat{p}_{\text{rework}} = Pr\{x > USL\} = 1 - Pr\{x < USL\} = 1 - \Phi \left( \frac{47 - 40}{1.756} \right) = 1 - \Phi(3.99) = 1 - 0.99997 = 0.00003 \text{ or 0.003\%.} \]

(e) 
First, center the process at 41, not 40, to reduce scrap and rework costs. Second, reduce variability such that the natural process tolerance limits are closer to, say, \( \sigma_x \approx 11.25 \).

5-11.

\( n = 6 \text{ items/subgroup}; \quad \sum_{i=1}^{50} x_i = 1000; \quad \sum_{i=1}^{50} S_i = 75; \quad m = 50 \text{ subgroups} \)

(a) 
\[ \overline{x} = \frac{\sum_{i=1}^{50} \overline{x}_i}{m} = \frac{1000}{50} = 20; \quad \overline{S} = \frac{\sum_{i=1}^{50} S_i}{m} = \frac{75}{50} = 1.50 \]
\[ \text{UCL}_x = \overline{x} + A_4\overline{S} = 20 + 1.287(1.50) = 21.93 \]
\[ \text{LCL}_x = \overline{x} - A_4\overline{S} = 20 - 1.287(1.50) = 18.07 \]
\[ \text{UCL}_s = B_4\overline{S} = 1.970(1.50) = 2.955 \]
\[ \text{LCL}_s = B_3\overline{S} = 0.030(1.50) = 0.045 \]

(b) 
natural process tolerance limits: \[ \overline{x} \pm 3\sigma_x = \overline{x} \pm 3 \left( \frac{\overline{S}}{c_4} \right) = 20 \pm 3 \left( \frac{1.50}{0.9515} \right) = [15.27, 24.73] \]

(c) 
\[ \hat{\sigma}_p = \frac{USL - LSL}{6\sigma_x} = \frac{+4.0 - (-4.0)}{6(1.58)} = 0.84 \text{, so the process is not capable.} \]

(d) 
\[ \hat{p}_{\text{rework}} = Pr\{x > USL\} = 1 - Pr\{x \leq USL\} = 1 - \Phi \left( \frac{23 - 20}{1.58} \right) = 1 - \Phi(1.90) = 1 - 0.97120 = 0.02880 \text{ or } 2.88\%. \]
\[ \hat{p}_{\text{scrap}} = Pr\{x < LSL\} = \Phi \left( \frac{15 - 20}{1.58} \right) = \Phi(-3.16) = 0.00078 \text{, or 0.078\%.} \]
Total = 2.88\% + 0.078\% = 2.958\%
5-11 continued
(e)
\[
\hat{p}_{\text{rework}} = 1 - \Phi\left(\frac{23 - 19}{1.58}\right) = 1 - \Phi(2.53) = 1 - 0.99432 = 0.00568 \text{, or } 0.568\%
\]
\[
\hat{p}_{\text{scrap}} = \Phi\left(\frac{15 - 19}{1.58}\right) = \Phi(-2.53) = 0.00568 \text{, or } 0.568\%
\]
Total = 0.568% + 0.568% = 1.136%
Centering the process would reduce rework, but increase scrap. A cost analysis is needed to make the final decision. An alternative would be to work to improve the process by reducing variability.

5-12.
(a)
MTB : Stat : Control Charts : Xbar/R Chart : Ex5-12ax1,x2,x3,x4,x5
Xbar/R Chart for Critical Dimension

(b)
MTB : Stat : Control Charts : Xbar/R Chart : Ex5-12bx1,x2,x3,x4,x5
Xbar/R Chart for Critical Dimension

Starting at Sample #21, the process average has shifted to above the UCL = 154.424.