Preparation for ECON-6570 Advanced Econometrics

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In preparation for the Advanced Econometrics course, please review fundamental skills with key topics used in the course: matrix algebra, exponents and logarithms, calculus, and probability. To review most of these topics, you could use any standard textbook on quantitative methods for economics, or appendices A and B of William H. Greene’s book *Econometric Analysis* (6th edition). Most importantly, though, look over the review materials I am providing for you. These are important to ensure that everyone has some basic background at the start of term.

Details are below.

**Matrix Algebra**

We will use matrix algebra throughout our PhD Econometrics course. Many of you know all of this already, but some people will need some review. If you don’t know part or all of the following, don’t panic, but do look it up ASAP. You can find some introductions on the internet (e.g., http://www.sosmath.com/matrix/matrix.html), or try a mathematics-for-economics book like Alpha Chiang’s *Fundamental Methods of Mathematical Economics*.

You should know:
- What is a **matrix**
- How to **add and subtract** matrices
- How to **multiply** a matrix by a constant and by another matrix
- How to **transpose** a matrix
- What are the following types of matrices: **square**, **symmetric**, **diagonal**, **identity**, **upper triangular**, **lower triangular**, **idempotent**
- What is an **inverse** of a matrix (don’t worry about how to compute it)
- What is the **rank** of a matrix, how to compute the **determinant** of a 2x2 matrix, and what you know about a matrix’s rank if its determinant equals zero

Using my matrix algebra practice sheets (see below), you should also learn how to compute derivatives and solve optimization problems in matrix form. For optimization, the practice sheets discuss unconstrained optimization, but it would help to understand equality-constrained optimization also (see a mathematics-for-economics book).

**Exponents and Logarithms and Calculus**

In addition to matrix algebra, you should have some basic skills for **exponents and logarithms** and for **differentiation**. If you are forgetting, please look these up. You should be able to simplify formulae involving exponents and logarithms, and you should be able to sketch the graphs of exponents and logarithms. You should understand what a derivative means, you should be able to compute derivatives for common formulae
including fractions, multiples, logarithms, and exponents; and you should be able to use
the chain rule for derivatives. Also, you should know what is an integral and how to
write (definite) integrals in formulae. (Solving integrals using formulas is of minor
importance for this course but it would help to learn to solve a few basic integral
formulas.)

**Probability**

Finally, the course will assume some basic knowledge of probability. You should
understand probability density functions, for one or multiple random variables, and the
associated cumulative density functions. You should understand and know how to
compute expectations including mean and variance, as well as conditional expectations.
You should understand joint probability density functions and how to compute
conditional probability density functions. For all of these you should be able to work
with both discrete and continuous random variables. If you have never learned these or
need review, you could use almost any standard book on the topic. Personally I like
Alvin W. Drake’s old textbook *Fundamentals of Applied Probability Theory* (McGraw-
Hill, 1967), which is dense but clear (see chapter 1 and especially chapter 2, and practice
by covering up the solutions to the examples and solving them yourself).

A background in statistics is not assumed, but would help. You could look at any
standard textbook on statistics, or Greene’s appendices C-D in *Econometric Analysis* (6th
edition). I will summarize but not prove key statistical theorems used in class.

**Practice Sheets**

I have prepared practice sheets with solutions, for much of the above: matrix algebra
including quadratic forms, exponents and logarithms, and derivatives. You will
definitely need to do the matrix algebra practice sheets; please complete this before our
first meeting (especially if you are weak on matrix algebra). The exponents & logarithms
and derivatives practice sheets are useful if you need to review those topics, and the
derivatives sheets are lengthier than we really need for this course (I use the derivatives
sheets for several courses).

**Some Key Facts**

The matrix algebra practice sheets point out the following facts, which you should learn
by heart for this course. For any matrices \( U \) and \( V \) for which the relevant additions,
multiplications, or inverses can be carried out:

\[
(U + V)' = U' + V' \\
(UV)' = V'U' \\
U^{-1}U = UU^{-1} = I \\
(UV)^{-1} = V^{-1}U^{-1}
\]

The matrix algebra sheets will also familiarize you with the following facts, which we
will use during the course and are worth memorizing:
\[
\frac{\partial (a'b)}{\partial b} = a
\]
\[
\frac{\partial (b'Ab)}{\partial b} = 2Ab \text{ if } A \text{ is symmetric}
\]

A scalar \( f(b) \) is minimized with respect to a K-dimensional vector \( b \) if
\[
\frac{\partial f(b)}{\partial b} = 0 \quad \text{and} \quad \frac{\partial^2 f(b)}{\partial b \partial b'} \text{ is positive definite (i.e. } \mathbf{x}' \frac{\partial^2 f(b)}{\partial b \partial b'} \mathbf{x} > 0 \quad \text{for all K-dimensional vectors } \mathbf{x} \neq 0)\]

The following facts from probability will be used often; it will help to understand these facts and why these are true:

- \( E[f(x)y \mid x] = f(x)E(y \mid x) \) “linearity of conditional expectations”
- \( E[E(y \mid x)] = E(y) \) “Law of Total Expectations”
- \( E[E(y \mid x, z) \mid x] = E(y \mid x) \) “Law of Iterated Expectations”

Try writing these out as integrals using the definition of an expectation or conditional expectation, then solving, so that you know why these are true. Then make sure you remember these facts and their names.