The Bigger Lego Workshop: How much would you pay to obtain more resources?

For the bigger Lego workshop problem, we formulated it as a linear program:

$$\text{maximize } 3x_1 + 2x_2 + 4x_3$$

subject to (BiggerWorkshop)

$$\begin{aligned} 2x_1 + x_2 + x_3 &\leq 200 \\ x_1 + x_2 + 2x_3 &\leq 200 \\ x_1 &\leq 40 \end{aligned}$$

$$x_1, x_2, x_3 \geq 0$$

and had determined its optimal solution to be $x_1 = 40, x_2 = 0, x_3 = 80$. Recall that x_1 represented the number of tees produced, x_2 the number of cake's produced, and x_3 was the number of I's produced. Further recall that the first constraint is the small bricks constraint, the second constraint is the large bricks constraint, and the final constraint is the tee upper bound constraint.

Three separate and independent opportunities have arisen that allow you to potentially obtain more resources:

- A friend has offered to supply you an additional small brick a week and is willing to negotiate a price with you.
- Your parents have offered to supply you an additional two big bricks a week and are willing to negotiate a price with you.
- An advertising specialist believes that they can increase the demand for the tees (thus increasing the tee selling upper bound). Their price is negotiable and is directly tied to the amount of new tee demand created. What are you willing to pay per unit of new tee demand?

In order to determine how to answer these separate questions, we will discuss the concept of shadow prices, how they can answer these questions, and how you can determine them.