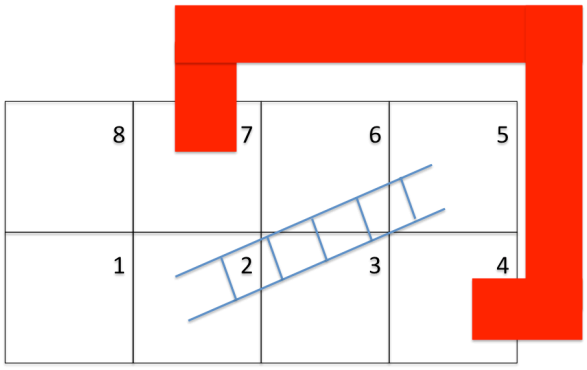
Chutes and Ladders: A Markov Chain Model

Chutes and Ladders is a board game that involves a 100 space board. Players start at Space 1 and try to reach Space 100. A spinner is spun and a player moves the number of spaces associated with the outcome of the spinn. Certain spaces are the base of a ladder and if a player lands on the base of the ladder, they get to follow the ladder up to the top of it (which is a higher numbered space than the bottom). Certain spaces are the tops of a chute (or slide) and if a player lands on the top of a slide, they move down to the bottom of it (which is a lowered number space than the top). In order to win the game, a player must *exactly* end up on 100 (so if they are on space 99, they need to spin a 1 to win). If a player cannot move the number they just spun, they will remain on their current space (so if they are on space 99 and spin a 2, they stay on 99).

We will focus on one player's journey through a simpler version of Chutes and Ladders. The board for this simpler version is below: there are eight spaces in this simpler version, one ladder from space 2 to space 5 and one chute from space 7 back to space 4. The spinner has integers 1, 2, 3, and 4 on it, which are each equally likely to occur from a single spin. We will throw in one wrinkle (which tends to happen when I play the game with my son since he likes having the game go on for as long as possible): when the player reaches space 8, they take one spin 'off' and move back to space 1 to start the game again.



We are interested in understanding the (1) percentage of time the player ends their turn on each space and (2) the expected number of turns from each space until we reach the end of the 'current' game (i.e., when do we end up on 8 again). We will formulate this board game as a Markov Chain to determine these values.