Example. (a) Solve the IVP using Euler’s method (RK1), explicit midpoint method (RK2M), explicit trapezoidal method (RK2T) and standard RK4:

\[ y' = e^{-t} - y, \quad 0 < t < 4, \quad y(0) = 0 \]

Use mesh spacings \( h = 4/N, \quad N = 20, 40, 80, 160, \) and compute the maximum error in the approximation for each grid. The exact solution is \( y(t) = te^{-t}. \) How does the error behave as \( h \) decreases?

(b) Repeat part (a) for the second-order equation

\[ y'' + 4y = \cos(3t), \quad 0 < t < 10, \quad y(0) = y'(0) = 0 \]

Use mesh spacings \( h = 10/N, \quad N = 200, 400, 800, 1600, \) and compute the maximum error in the approximation for each grid. The exact solution is \( y(t) = \left(\frac{2}{5}\right) \sin(t/2) \sin(5t/2). \) How does the error behave as \( h \) decreases? Compare the computational performance of the RK methods in terms of the number of evaluations of the slope function.