1. Use a suitable eigenfunction expansion to determine the temperature $u(r, t)$ satisfying

$$\frac{\partial u}{\partial t} = \frac{k}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) + Q(r, t), \quad a < r < b, \quad t > 0$$

with initial condition $u(r, 0) = f(r)$ and boundary conditions $u(a, t) = u(b, t) = 0$. Is it valid to differentiate the eigenfunction expansion term-by-term?

2. Text exercises 8.5.2 and 8.5.3, pages 364–365.

3. Text exercise 8.5.4, part (c) only, page 365.

4. Consider the Poisson problem

$$u_{xx} + u_{yy} = Q(x, y), \quad 0 < x < L, \quad 0 < y < H$$

with

$$u_x(0, y) = u_x(L, y) = 0, \quad u(x, 0) = u(x, H) = 0$$

(a) Find the solution in terms of an eigenfunction expansion of the form

$$u(x, y) = \sum_n A_n(y) \phi_n(x)$$

(b) Find the solution in terms of an eigenfunction expansion of the form

$$u(x, y) = \sum_\lambda C_\lambda \Phi_\lambda(x, y)$$

where $\Phi_\lambda(x, y)$ are suitable eigenfunctions of a Helmholtz equation.

5. Consider the boundary-value problem

$$(r^2 R')' - 6R = Q(r), \quad R(0) \text{ bounded}, \quad R(1) = 0$$

where $Q(r)$ is a smooth forcing function. Use variation of parameters to find the solution for $R(r)$. Write the solution in the form

$$R(r) = \int_0^1 G(r, \bar{r})Q(\bar{r}) \, d\bar{r}$$

Give $G(r, \bar{r})$ explicitly and verify (perhaps using L'Hôpital's rule) that $R(r)$ is bounded as $r \to 0$. 
