Example. The temperature $u(x,t)$ in a bar of length $L$ satisfies the heat flow problem

$$u_t = ku_{xx}, \quad 0 < x < L, \quad t > 0$$

with initial condition $u(x,0) = f(x)$ and Dirichlet boundary conditions $u(0,t) = u(L,t) = 0$. The length $L$, diffusivity $k$ and function $f(x)$ are considered to be known.

Recall the general solution satisfying the PDE and homogeneous boundary conditions.

$$u(x,t) = \sum_{n=1}^{\infty} C_n \exp \left[-\left(\frac{n\pi}{L}\right)^2 kt\right] \sin\left(\frac{n\pi x}{L}\right)$$

Determine the solution for the cases:

(a) $f(x) = 1$  \hspace{1cm} (b) $f(x) = x(L - x)$