**Example.** The temperature $u(r, \theta)$ in a 2D disk satisfies the steady heat flow problem

$$\nabla^2 u = \frac{1}{r} (ru_r)_r + \frac{1}{r^2} u_{\theta \theta} = 0, \quad 0 < r < a, \quad |\theta| \leq \pi$$

with the Dirichlet boundary condition $u(a, \theta) = f(\theta)$. The radius $a$ and function $f(\theta)$ are considered to be known.

(a) Use separation of variables to determine separated equations (ODEs).

(b) Identify the eigenvalue problem and find all non-trivial solutions.

(c) Use superposition to determine the general solution satisfying the PDE and the boundary conditions in part (a).

(d) Evaluate the coefficients in the expansions of the general solution and evaluate the solution at $r = 0$. 