**Example.** Consider the heat flow problem

\[ u_t = ku_{xx}, \quad 0 < x < L, \quad t > 0, \]

with

\[ u(x, 0) = f(x), \quad u(0, t) = u(L, t) = 0, \]

where \( k > 0 \) is a given constant and \( f(x) \) is a given piecewise smooth function. Assuming \( u_t, u_x \) and \( u_{xx} \) are continuous functions (and the one-sided limits of \( u(x, t) \) at \( x = 0, L \) exist for \( t > 0 \) and are both equal to zero), then determine a solution of the form of an *eigenfunction expansion*

\[ u(x, t) = \sum_{n=1}^{\infty} B_n(t) \sin \left( \frac{n\pi x}{L} \right) \]

where \( B_n(t) \) are time-dependent coefficients.