1. Consider the differential equation \( y' = 3 + 2y \). Sketch a direction field for the differential equation and determine the behavior of \( y \) as \( t \to \infty \). If the behavior depends on the initial value of \( y \) at \( t = 0 \), then describe that dependency.

Let \( y(0) = y_0 \). If \( y_0 > -\frac{3}{2} \), then \( y \to \infty \) as \( t \to \infty \). If \( y_0 < -\frac{3}{2} \), then \( y \to -\infty \) as \( t \to \infty \). If \( y_0 = -\frac{3}{2} \), then the solution is constant and \( y \to -\frac{3}{2} \) as \( t \to \infty \).

2. A population \( y(t) \) of field mice grows at a rate proportional to the amount present at any given time so that it satisfies the differential equation \( y' = ky \) for some positive rate constant \( k \).

(a) If the population doubles in 3 months, then determine the value of the rate constant in the differential equation. Be sure to provide the correct units for the rate constant.

From the problem statement, with time in months:

\[
y' = ky, \quad y(0) = y_0, \quad y(3) = 2y_0
\]

\[
\frac{y'}{y} = k
\]

\[
\ln|y| = kt + c
\]

\[
y = Ce^{kt}
\]

Applying the initial condition

\[
y(0) = C = y_0
\]

Applying the second condition

\[
y(3) = y_0e^{3k} = 2y_0
\]

\[
k = \frac{\ln(2)}{3} \approx 0.2310 \text{ month}^{-1}
\]

(b) Suppose the rate constant in the differential equation is \( k = (1/10) \) days\(^{-1} \). Find the time it takes for the population to become four times its original value.
\[ y_0 e^{1/10 T} = 4 y_0 \quad \Rightarrow \quad T = 10 \ln(4) \approx 13.863 \text{ days} \]

3. Solve the initial-value problem

\[ y' + 3y = 1, \quad t > 0, \quad y(0) = 0 \]

First, find the general solution to the ODE.

\[ y' = -3 \left( y - \frac{1}{3} \right) \]

\[ \frac{y'}{y - \frac{1}{3}} = -3 \]

\[ \ln \left| y - \frac{1}{3} \right| = -3t + c \]

\[ y = \frac{1}{3} + Ce^{-3t} \]

Next, apply the initial condition.

\[ y(0) = \frac{1}{3} + C = 0 \]

The solution is then \[ y = \frac{1}{3} - \frac{1}{3} e^{-3t} \]