Please answer all of the following questions. The “starred” problems will be graded while the remaining problems will be checked for completeness. Staple your work to this sheet of paper and indicate your answers clearly. Don’t forget your name and please circle your recitation time below.

Tuesday: 2–3pm       Tuesday: 3–4pm       Friday: 2–3pm       Friday: 3–4pm

Starred Problems:

1. Let

\[ A = \begin{bmatrix} 2 & 3 & 2 \\ 4 & 1 & 1 \\ -4 & -2 & -1 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 4 \\ -3 \end{bmatrix} \]

Show that the column vectors of \( A \) are linearly independent, and use row reduction followed by backwards substitution to find a vector \( x \) such that \( Ax = b \).

2. Consider the constant-coefficient system

\[ x' = \begin{bmatrix} 4 & -3 \\ 6 & -5 \end{bmatrix} x \]

Find the general solution and describe its behavior in the phase plane.

3. Solve the initial-value problem

\[ x' = \begin{bmatrix} 1 & 1 \\ -3 & 5 \end{bmatrix} x, \quad x(0) = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \]

Describe the behavior of the solution in the phase plane as \( t \to \infty \).

Non-Starred Problems:

4. Section 7.2 (pp. 376–378) 2, 4, 8(a,b), 10.

5. Section 7.3 (pp. 388–390) 3, 7, 16, 17.

6. Section 7.5 (pp. 405–407) 4(a), 15.