Diff. Eqns.
Problem Set 6
Due: Monday, 3/26/18
Name: ________________________________

Please answer all of the following questions. The “starred” problems will be graded while the remaining problems will be checked for completeness. Staple your work to this sheet of paper and indicate your answers clearly. Don’t forget your name and please circle your recitation time below.

Tuesday: 2–3pm       Tuesday: 3–4pm       Friday: 2–3pm       Friday: 3–4pm

Starred Problems:

1. Find all real eigenvalues $\lambda$ and eigenfunctions $y(x)$ of the given problems. Consider the cases (i) $\lambda > 0$, (ii) $\lambda = 0$ and (iii) $\lambda < 0$ for each problem.

   (a) $y'' + \lambda y = 0$, $0 < x < 2$, $y'(0) = 0$, $y(2) = 0$
   (b) $y'' - \lambda y = 0$, $0 < x < 3\pi$, $y(0) = 0$, $y(3\pi) = 0$

2. Let

   $$f(x) = \begin{cases} 
   1 & \text{if } -1 \leq x < 0 \\
   x - 1 & \text{if } 0 \leq x < 1 
   \end{cases}$$

   Find $F(x)$, the Fourier series of $f(x)$ with $L = 1$. Sketch $F(x)$ for the interval $-4 \leq x \leq 4$.

3. Let

   $$f(x) = 2 - x, \quad \text{for } 0 \leq x \leq 2$$

   The Fourier sine series of $f(x)$ is $S(x)$ and the Fourier cosine series is $C(x)$. Find $S(x)$ and $C(x)$ with $L = 2$, and sketch the two series for $-6 \leq x \leq 6$.

Non-Starred Problems:

4. Section 10.2 (pp. 605–607) 2, 3, 10, 15.

5. Section 10.3 (pp. 612–614) 2, 17.

6. Section 10.4 (pp. 620–623) 1, 4, 7, 15.