1. Find the general solution of the nonhomogeneous differential equation

\[ y'' - y' - 2y = e^{2t} + \cos t \]
2. A spring is stretched 0.2 m by a force of 3 N (i.e. 3 newtons). A 3 kg mass is hung from the spring and also attached to a device which exerts a viscous damping force of 5 N when the velocity of the mass is 2 m/s. There is no external forcing on the system. The mass is set into motion from its equilibrium position by an initial upward velocity of 4 m/s.

(a) Determine the spring constant $k$ and the damping coefficient $\gamma$ of the system.

(b) Give the initial-value problem whose solution specifies the downward displacement of the mass, $u(t)$, as a function of time $t$. You need not solve the problem for $u(t)$. 
2. (c) A different mass-spring system with periodic forcing satisfies

\[ 2u'' + 9u = 5 \cos 2t \]

State which graph below best describes the behavior of \( u(t) \) for this system. (You need not solve the differential equation for \( u(t) \), but briefly explain your choice.)

Plot A

Plot B

Plot C

Plot D
3. Solve the boundary-value problem for $v(x)$ or show that a solution does not exist.

\[ v'' + 4v = 1 + x - x^2, \quad v(0) = 9, \quad v(\pi) = 3. \]
4. Let
\[ f(x) = \begin{cases} 
  x & \text{for } 0 \leq x < 1 \\
  0 & \text{for } 1 \leq x \leq 2
\end{cases} \]

(a) Find \( C(x) \), the Fourier cosine series of \( f(x) \) with \( L = 2 \).

(b) Sketch \( C(x) \) for the interval \(-6 \leq x \leq 6\) on the graph below. (Indicate where the series converges at discontinuities.)
5. Consider the nonhomogeneous differential equation

\[ t^2 y'' + 2ty' - 6y = 2t^3 + t^2, \quad t > 0 \]

(a) Show that \( y_1(t) = t^2 \) and \( y_2(t) = t^{-3} \) are independent solutions of the corresponding homogeneous equation.

(b) Use variation of parameters to find a particular solution of the nonhomogeneous equation.