Instructions:
• Please answer all 5 problems.
• No books, notes, laptops or calculators permitted.
• One crib sheet is allowed.

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1. (a) Solve

$$t^3 y' + 4t^2 y = e^t, \quad y(1) = 0, \quad t > 0$$
1. (b) Solve the initial-value problem

\[ \frac{t y'}{t^2 + 1} + y^3 = 0, \quad y(1) = -\frac{1}{2} \]

Write your solution in explicit form, i.e. give \( y \) explicitly as a function of \( t \).
2. A certain population \( y(t) \) of insects is well-modeled by a logistic equation of the form

\[
\frac{dy}{dt} = (r - ay) y,
\]
where \( r \) and \( a \) are positive constants.

(a) Let \( f(y) = (r - ay) y \). Sketch \( f(y) \) versus \( y \) for the case \( r = 2 \) and \( a = 1 \). Use the sketch to determine all equilibrium points of the differential equation and to label them as stable or unstable. (Give brief reasons for your choices.)
2. (b) Recall again the population model

\[ \frac{dy}{dt} = (r - ay)y, \quad \text{where } r \text{ and } a \text{ are positive constants.} \]

It is known that (i) the population increases at a rate of 50 insects per hour when the population is equal to 100 insects and (ii) the population settles to 600 insects after a very long time. Find \( r \) and \( a \) for this case.
3. Find general solutions for the following second-order equations:

(a) \[ 25y'' - 30y' + 9y = 0, \]
(b) \[ 2y'' + 2y' + 3y = 0 \]
4. Find an interval of $t$ such that a unique solution of the initial-value problem is guaranteed to exist

$$(t^2 - 3)y'' + \frac{ty'}{t+1} + e^ty = 0, \quad y(0) = 0, \quad y'(0) = -2$$

You need not find the solution, but you should give reasons for your choice of an interval of $t$. 

5. A student takes out a loan for $5000 just before coming to RPI to help her parents pay the cost of tuition. The annual interest rate on the loan is fixed at 5% which is compounded continuously.

(a) Determine how much the student owes after 4 years at RPI.

(b) Assume the student owes $N$ dollars after 4 years at RPI. The student then gets a good job and begins paying off the loan at a continuous rate of $600 per year. (The interest rate is still 5%.) Determine how long it will take for the student to pay off the loan after leaving RPI.