**Example.** Consider the population model

\[ y' = -3 \left( 1 - \frac{y}{2} \right) \left( 1 - \frac{y}{3} \right) y \]

(a) Find all critical (equilibrium) values of \( y \) and determine whether they are asymptotically stable or unstable.

(b) Suppose \( y(0) = y_0 > 0 \). Consider the behavior of \( y(t) \) as \( t \to \infty \). How does the behavior depend on the value of \( y_0 \)?

\[ y' = F(y), \quad F = -3 \left( 1 - \frac{y}{2} \right) \left( 1 - \frac{y}{3} \right) y = \text{cubic polynomial} \]

\[ F = 0 \text{ when } y = 0, 2, 3 \]

Plot:

\[ F < 0 \quad F > 0 \quad F < 0 \]

\[ F > 0 \]

\[ F(0) = 0, \quad F'(0) < 0 \Rightarrow y_c = 0 \text{ is stable} \]

\[ F(2) = 0, \quad F'(2) > 0 \Rightarrow y_c = 2 \text{ is unstable} \]

\[ F(3) = 0, \quad F'(3) < 0 \Rightarrow y_c = 3 \text{ is stable} \]
Continued.

b) Behavior for $y(0) = y_0$

Direction field plot (for example)

If $0 < y_0 < 2$, then $y(t) \to 0$ as $t \to \infty$

If $y_0 > 2$, then $y(t) \to y_c = 3$ as $t \to \infty$
Continued.