**Example.** John opens an IRA at age 25, contributes at a continuous rate of $2000 per year for 10 years, but makes no additional contributions after that. Susan waits until age 35 to open an IRA and contributes at a continuous rate of $2000 per year for 30 years. There is no initial investment in either case.

(a) Find an expression for the amount in each person’s IRA at age 65 assuming a fixed interest rate.

(b) Which investment strategy is best?

\[ P_J(t) = \text{bucks in John's IRA} \]
\[ P_S(t) = \text{bucks in Susan's IRA} \]
\[ t = \text{time (yr.)} \]

**Assumption:**

\[ B' = k B + P \]
\[ k = \text{interest rate} \]
\[ P = \text{rate of payment} \]

**John**

\[ B_J' = k B_J + 2000 \]
\[ = k \left( B_J + \frac{2000}{k} \right) \]
\[ \frac{B_J'}{B_J + \frac{2000}{k}} = k \]

\[ B_J(10) = 0 \]
\[ t = 0 \Rightarrow \text{age} = 25 \]
Continued.

\[ \ln \left( \frac{B_5}{K} \right) + \frac{2000}{K} = kt + C \]

\[ B_5(t) = -\frac{2000}{K} + Ce^{kt} \quad \text{I.C.: } B_5(0) = 0 \Rightarrow C = -\frac{2000}{K} \]

\[ B_5(10) = \frac{2000}{K} \left( e^{10k} - 1 \right) = A \]

Next 30 yrs.

\[ B_5' = k B_5 \quad B_5(10) = A \]

So in.

\[ B_5(t) = C e^{kt} \quad \text{I.C. } \Rightarrow C = Ae^{-k10} \]

At age 65 \( \Rightarrow t = 40 \)

\[ B_5(40) = Ae^{30k} = \frac{2000}{K} \left( e^{10k} - 1 \right) e^{30k} \]

\[ \underline{Susan} \]

\[ B_5' = k B_5 + 2000 \quad B_5(10) = 0 \]

\[ B_5(t) = -\frac{2000}{K} + Ce^{kt} \quad \text{I.C. } \Rightarrow C = -\frac{2000}{K} e^{-10k} \]

\[ B_5(t) = \frac{2000}{K} \left( e^{k(t-10)} - 1 \right) \]

At 65 \( \Rightarrow t = 40 \)

\[ B_5(40) = \frac{2000}{K} \left( e^{30k} - 1 \right) \]
Continued.

\( B^{(4 \omega)} \)

John

Susan

\( \approx D \omega \)