Example. (a) A sample of thorium-234 decays from 100 mg to 82.04 mg in one week. Determine the rate constant and the time it takes for the sample to reduce to one-half the original.

(b) Suppose thorium-234 is added to the sample at a (continuous) rate of 0.3 mg/day. Find an expression for the mass of the sample at any given time.

\[ a) \ \text{Let} \ t = \text{time (day)} \]
\[ Q(t) = \text{mass of Thorium-234 (mg)} \]

Model: Assume
\[ Q' = -kQ \quad k > 0 \text{ is the rate constant} \]
\[ Q(0) = 100 \]

Solution:
\[ Q(t) = Ce^{-kt} \]

I.C. \[ Q(0) = 100 = C \]
\[ Q(t) = 100e^{-kt} \]

Given
\[ Q(7) = 82.04 = 100e^{-k \cdot 7} \]
Continued.

\[ K = -\frac{1}{7} \ln \left( 0.8204 \right) = 0.028 \text{ day}^{-1} \]

Half-life: \( Q(T) = 50 \)

\[ 50 = 100e^{-kT} \]

\[ \Rightarrow T = -\frac{1}{k} \ln \left( \frac{1}{2} \right) = \frac{1}{k} \ln(2) \]

b) Assume

\[ Q' = -kQ + P \quad , \quad P = 0.3 \text{ mg day}^{-1} \]

Solve

\[ Q(t) = \frac{P}{k} + Ce^{-kt} \]

\[ Q(0) = 100 \quad , \quad Q(0) = \frac{P}{k} + C = 100 \]

\[ C = 100 - \frac{P}{k} \]

\[ \Rightarrow Q(t) = 100e^{-kt} + \frac{P}{k} \left( 1 - e^{-kt} \right) \]