Example. A mass weighing 2 lb stretches a spring 3 inches. An oscillation is initiated in the mass-spring system by a further displacement of the mass equal to 2 inches and by a downward velocity of 1/2 ft/sec. Find the frequency and amplitude of the oscillation.

Spring: \[ k = \frac{mg}{x} = \frac{2 \text{ lb}}{\frac{1}{4} \text{ ft}} = 8 \text{ lb/ft} \]

Mass: \[ mg = 2 \text{ lb} \Rightarrow m = \frac{2 \text{ lb}}{32 \text{ ft/ sec}^2} = \frac{1}{16} \text{ slugs} \]

\[ \frac{1}{16} \ddot{u} + 8u = 0 \]

IC: \[ u(0) = \frac{1}{4} \text{ ft}, \quad u'(0) = \frac{1}{2} \text{ ft/sec} \]

Set \[ u = e^{rt} \Rightarrow \frac{r^2}{16} + 8 = 0 \]

\[ r^2 = -8 \cdot 16 \]

\[ r = \pm i \sqrt{128} \]

\[ \omega_0 = \text{freq.} \]

So ln. \[ u = C_1 \cos(\omega_0 t) + C_2 \sin(\omega_0 t) \]

IC: \[ u(0) = C_1 = \frac{1}{4} \]

\[ u'(0) = \omega_0 C_2 = \frac{1}{2} \quad C_2 = \frac{1}{2\omega_0} \]
Continued.

\[ u(t) = \frac{1}{2} \cos(w_0 t) + \frac{1}{2w_0} \sin(w_0 t) \]

\[ = A \cos(w_0 t - \delta) \]

\[ = A \left( \cos(w_0 t) \cos \delta + \sin(w_0 t) \sin \delta \right) \]

\[ = A \cos \delta = \frac{1}{6}, \quad A \sin \delta = \frac{1}{2w_0} \]

\[ A^2 \cos^2 \delta + A^2 \sin^2 \delta = \frac{1}{36} + \frac{1}{4w_0^2} \]

\[ A^2 = \sqrt{\frac{1}{36} + \frac{1}{4w_0^2}} = \omega \rho_1. \]