Example. Let $y(x)$ solve the homogeneous ODE and homogeneous BCs

$$y'' + \lambda y = 0, \quad y(0) = 0, \quad y(\pi) = 0$$

Find all values of the real parameter $\lambda$ (eigenvalues) such that nontrivial solutions (eigenfunctions) exist. In particular, examine the three cases (a) $\lambda > 0$, (b) $\lambda = 0$ and (c) $\lambda < 0$. 
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