Please answer all of the following questions. The “starred” problems will be graded while the remained problems will be checked for completeness. Staple your work to this sheet of paper and indicate your answers clearly. Don’t forget your name and please circle your recitation time below.

Tuesday: 2–3pm Tuesday: 3–4pm Friday: 2–3pm Friday: 3–4pm

Starred Problems:

1. Solve the initial-value problem

\[ \mathbf{x}' = \begin{bmatrix} 7 & 9 \\ -6 & -8 \end{bmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{bmatrix} 4 \\ -3 \end{bmatrix} \]

Describe the behavior of the solution as \( t \to \infty \).

2. Consider the system

\[ \mathbf{x}' = \begin{bmatrix} 1 & -1 \\ 5 & -3 \end{bmatrix} \mathbf{x} \]

Find the general solution of the system and express this solution in terms of real-valued functions.

3. Consider the mass-spring-damper equation

\[ m\dddot{u} + \gamma \dot{u} + ku = 0 \]

(a) Let \( x_1 = u \) and \( x_2 = u' \) and write the second-order equation for \( u(t) \) as a first-order system for \( \mathbf{x}(t) \).

(b) Find the general solution (in terms of real-valued functions) for the linear system in part (a) for the cases: (i) \( m = 1, \gamma = 4, k = 3 \), and (ii) \( m = 1, \gamma = 4, k = 5 \).

(c) Sketch the behavior of solutions in the \( x_1-x_2 \) phase plane for the two cases in part (b).

Non-Starred Problems:

4. Section 7.5 (pp. 405–407) 4(a), 15, 16, 24(a).

5. Section 7.6 (pp. 417–420) 2(a), 3(a), 9, 15.

6. Section 9.1 (pp. 505–508) 1(a,b), 2(a,b), 5(a,b).