Please answer all of the following questions. The “starred” problems will be graded while the remained problems will be checked for completeness. Staple your work to this sheet of paper and indicate your answers clearly. Don’t forget your name and please circle your recitation time below.

Tuesday: 2–3pm      Tuesday: 3–4pm      Friday: 2–3pm      Friday: 3–4pm

Starred Problems:

1. Solve the boundary-value problems for $y(x)$, or else show that no solutions exist.

   (a) $y'' + y' - 6y = 0, \quad 0 \leq x \leq 1, \quad y(0) = 2, \quad y(1) = -4$

   (b) $y'' + 9y = x + e^x, \quad 0 \leq x \leq \pi, \quad y(0) = 0, \quad y(\pi) = 0$

2. Find the eigenvalues $\lambda$ and eigenfunctions $y(x)$ of the given boundary-value problems.
   (You may assume that all eigenvalues are real.)

   (a) $y'' + \lambda y = 0, \quad 0 \leq x \leq 3, \quad y(0) = 0, \quad y'(3) = 0$

   (b) $y'' - \lambda y = 0, \quad 0 \leq x \leq 2\pi, \quad y'(0) = 0, \quad y(2\pi) = 0$

3. Let

   $$f(x) = \begin{cases} 2 & \text{if } -1 \leq x < 0 \\ 2 - 2x & \text{if } 0 \leq x < 1 \end{cases} \quad \text{and} \quad f(x + 2) = f(x)$$

   Sketch the graph of $f(x)$ for $-2 \leq x \leq 4$ and find its Fourier series.

4. Let

   $$f(x) = -x, \quad -1 \leq x < 1, \quad \text{and} \quad f(x + 2) = f(x)$$

   Sketch the graph of $f(x)$ for $-2 \leq x \leq 4$ and find its Fourier series.

Non-Starred Problems:

5. Section 10.1 (pp. 595–596) 11, 16, 21. (See equation (2) in Section 5.4 and the discussion of it that follows for problem 11.)

6. Section 10.2 (pp. 605–607) 1, 3, 6, 14, 15.