1. (a) Solve the linear system or else show that there is no solution

\[ \begin{align*}
x_1 + 2x_2 - x_3 &= 1 \\
2x_1 + x_2 + x_3 &= 1 \\
x_1 - x_2 + 2x_3 &= 1
\end{align*} \]
1. (b) Determine whether the following three vectors are linearly independent. If they are not linearly independent, then show that one vector can be written as a linear combination of the other two vectors.

\[ \mathbf{x}_1 = (2, 1, 0), \quad \mathbf{x}_2 = (0, 1, 0), \quad \mathbf{x}_3 = (-1, 2, 0) \]
2. Find the solution $\mathbf{x}(t)$ of the initial-value problem

$$\mathbf{x}' = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$
3. Consider the linear system $x' = Ax$, where $A$ has eigenvalues $(\lambda_1, \lambda_2)$ and eigenvectors $(z_1, z_2)$ given by

$$
\lambda_1 = -1 + 3i, \quad \lambda_2 = -1 - 3i, \quad z_1 = \begin{bmatrix} 1 \\ -1 - i \end{bmatrix}, \quad z_2 = \begin{bmatrix} 1 \\ -1 + i \end{bmatrix}
$$

(a) Find the general solution of the system in terms of real-valued functions.

(b) The plots on the next page show trajectories and direction fields for different linear systems. Choose the one that best matches the behavior of the system above. Give reasons for your choice.
4. Find all equilibrium (i.e. critical) points of the following equations

(a) \[ \begin{align*}
x' &= x(1 - 3x - y) \\
y' &= y(2 - y - 2x)
\end{align*} \]

(b) \[ \begin{align*}
x' &= x \ln y \\
y' &= 2 - e^y - x^2
\end{align*} \]
5. Consider the differential equations

\[ \frac{dx_1}{dt} = -x_1 + 2x_2, \quad \frac{dx_2}{dt} = 4x_1 + x_2 \]

(a) Find the general solution.

(b) Classify the solution as to type (stable node, saddle, etc) and illustrate the behavior by sketching trajectories in the \((x_1, x_2)\) phase plane.