Problem Set 5 Solutions

Due: October 19, 2015

1 Problem 1

\[ mg = 4 \text{lb}, d = .5 \text{ft} \implies 4 \text{lb} = k \cdot 5 \text{ft} \implies k = 8 \text{lb/ft} \] (1)

\[ \gamma = 0 \text{ no dampening}, m = 4 \text{lb}/32 \text{ft}/s^2 = 1/8 \text{lbs}^2/\text{ft} \] (2)

\[ u(0) = -1/3 \text{ft}, u'(0) = 2 \text{ft/sec} \] (3)

\[ \omega_0^2 = k/m = 64 \mu u'' + ku = 0 \] (4)

\[ u(t) = A \cos(\omega_0 t) + B \sin(\omega_0 t) \] (5)

\[ u(0) \implies A = -1/3 \] (6)

\[ u'(0) = 2 = B \omega_0 \implies B = 1/4 \] (7)

\[ f = \frac{\omega_0}{2\pi} = 4/\pi \text{hz} \approx 1.273 \text{hz} \] (8)

\[ T = 2\pi/\omega_0 = \pi/4 \text{s} \approx .785 \] (9)

\[ \text{Amp} = R = \sqrt{A^2 + B^2} = \sqrt{\frac{1}{3^2} + \frac{1}{4^2}} = \frac{5}{12} \] (10)

2 Problem 2

\[ 2N = 5 \text{cmN} \implies k = .4 \text{N/cm} = 40 \text{N/m} \] (12)

\[ 2N = \gamma 4 \text{m/s} \implies \gamma = .5 \text{Ns/m} \] (13)

\[ m = 3 \text{kg}, \mu = \frac{\sqrt{4km - \gamma^2}}{2m} = \sqrt{479.75}/6 \approx 3.651 \] (14)

\[ \mu u'' + \gamma u' + ku = 0 \] (15)

\[ u(0) = 3, u'(0) = -5 \] (16)

\[ u(t) = e^{-\gamma t/2m} (A \cos(\mu t) + B \sin(\mu t)) \] (17)

\[ u(t) \approx e^{-t/12}(3 \cos(3.651 t) - 1.301 \sin(3.651 t)) \] (18)
3 Problem 3

3.1 Part a

\[ \omega_0 = 3 \] (19)

\[ u(t) = A \cos(\omega_0 t) + B \sin(\omega_0 t) + \frac{F_0}{m(\omega_0^2 - \omega^2)} \cos \omega t \] (20)

\[ u(t) = -\frac{F_0}{m(\omega_0^2 - \omega^2)} \cos(\omega_0 t) + \frac{1}{\omega_0} \sin(\omega_0 t) + \frac{F_0}{m(\omega_0^2 - \omega^2)} \cos \omega t \] (21)

\[ u(t) = -\frac{1}{3^2 - \pi^2} (\cos(3t) - \cos(\pi t)) + \frac{1}{3} \sin(3t) \] (22)

\[ u(t) = \frac{2}{8696} \sin((\pi - 3)t/2) \sin((3 + \pi)t/2) + \frac{1}{3} \sin(3t) \] (23)

3.2 Part b

\[ \omega_0 = 4 = \omega \implies \text{resonance} \] (24)

\[ u(t) = A \cos(\omega t) + B \sin(\omega t) + \frac{F_0}{2m\omega} t \sin(\omega t) \] (25)

\[ u(t) = \frac{1}{4} \sin(4t) + \frac{1}{4} t \sin(4t) \] (26)

\[ u(t) = \frac{1}{4} (1 + t) \sin(4t) \] (27)