Numerical Computing
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Problem Set 8

Due:
Tuesday, 11/24/98

1. Exercise 9.2, page 296.

2. Consider the ODE $y' = -\dfrac{y}{1+t}$.

(a) Sketch by hand the direction field for the ODE for the region $0 \le t \le 4$ and $-2 \le y \le 2$.

(b) Find an expression for the integral curves of the ODE parameterized by a constant C.

(c) Find the exact solution of the ODE subject to the initial condition $y(0) = -2$. Sketch this solution in the direction field plot in part (a).

(d) Use Euler's method to approximate the solution in part (c) for $0 \le t \le 4$ using $h = .4$, $.2$, and $.1$. Plot the approximate solutions and the exact solution on the same graph. Compute the global error, $\max|w_i - y(t_i)|$, for each approximate solution. How does the error depend on h?

3. Consider the ODE $y' = f(t, y)$, where $f = -12(y-1)$, and the numerical methods

$$w_{i+1} = w_i + hf(t_i, w_i), \qquad \text{Euler's method}$$
$$w_{i+1} = w_i + hf(t_{i+1}, w_{i+1}), \qquad \text{Backward Euler's method}$$

(a) Find the exact solution of the ODE with initial condition $y(0) = 0$. Describe in words the behavior of the solution. Sketch the solution $y(t)$.

(b) Find numerical solutions of the ODE with initial condition $y(0) = 0$ for the interval $0 \le t \le 1$ using the two Euler methods given above. (In the case of Backward Euler you can use a little algebra to solve the difference equation for w_{i+1}.) Experiment using $h = 0.1$ and $h = 0.2$. Comment on the behavior of the numerical solution and the stability of the methods for this equation.

4. The pendulum equation $\theta'' + \sin\theta = 0$ is equivalent to the first-order system

$$y_1' = y_2$$
$$y_2' = -\sin y_1$$

(a) Write a Matlab script or function that solves this system using the Midpoint method. Plot your computed $\theta = y_1$ (the angular displacement of the bob) versus t on the interval $[0, 10]$ for the cases (i) $y_1(0) = 0$, $y_2(0) = 1.5$; and (ii) $y_1(0) = 0$, $y_2(0) = 2.1$. (Hint: You may use the sample M-file `euler.m` in my public directory as a guide.)

(b) Repeat part (a) using Matlab's ODE solver `ode23`.