

CALCULUS 2 SPRING 2009 FINAL EXAM PRACTICE PROBLEMS

**Note:** The problems below are intended to help prepare for the conventional portion of the final exam. If you wish to ask about skills problems at the review class, you may do so. There is no guarantee that every topic we have discussed is covered in these problems. For a complete list of exam coverage, see [www.rpi.edu/~schmid](http://www.rpi.edu/~schmid)

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1. Evaluate each of the following integrals:

a.)  $\int x^{\frac{3}{2}} \ln x \, dx$

b.)  $\int \frac{4x^2 + x - 2}{x^3 - x^2} \, dx$

2. Evaluate the improper integral:  $\int_0^1 x \ln x \, dx$

3. Consider the parametric equations  $x = \frac{\sin^2 t}{2}$ ,  $y = \cos t$

a.) Find a corresponding cartesian equation for these parametric equations.

b.) Sketch the path of a particle whose position is given by these parametric equations for the time interval  $3\pi/2 \leq t \leq 2\pi$  and indicate the direction in which the path is swept out.

c.) Find an equation for the tangent line at the point  $P(3/8, 1/2)$ .

4. a.) Sketch the graph of the polar equation  $r = \sin 5\theta$ .

b.) Find the area inside one leaf of this leaf-curve.

5. Determine whether each of the following infinite series converges absolutely, converges conditionally or diverges. Justify your answers.

a.)  $\sum_{k=0}^{\infty} \frac{(-1)^k}{k+10}$       b.)  $\sum_{k=2}^{\infty} \frac{k^2+1}{k^3-2}$       c.)  $\sum_{k=1}^{\infty} \frac{(-1)^k 3^k}{k^2 k!}$

6. Find the Maclaurin Series for  $f(x) = \ln(3+x)$  and find its radius of convergence.

7. Consider the two planes  $P_1 : x + 3y - 4z = 5$  and  $P_2 : 3x - y - z = 2$ , and the line with symmetric equations  $L : \frac{x-2}{1} = \frac{y-1}{2} = \frac{z-1}{2}$ .

a.) Find the point where  $L$  intersects  $P_1$ .

b.) Show that the two planes are not parallel.

c.) The two planes intersect in a line. Find parametric equations for their line of intersection.

8. A cannonball is launched off the front of a ship 80 feet above sea level with an initial speed of 128 feet per second at an angle of 30 degrees. Find the total **horizontal** distance the cannonball travels before hitting the water.

9. The space curves corresponding to the two vector-valued functions

$$\mathbf{r}_1(t) = \sin(t)\vec{\mathbf{i}} + (t+1)\vec{\mathbf{j}} + \cos(t)\vec{\mathbf{k}}$$

$$\mathbf{r}_2(t) = t^3\vec{\mathbf{i}} + e^t\vec{\mathbf{j}} + (1-t^2)\vec{\mathbf{k}}$$

intersect at the point  $P(0, 1, 1)$ . Find the angle between the two curves' tangent lines at this point of intersection.

10. Consider the function of two variables  $f(x, y) = \ln(x^2 - y^2)$ .

a.) Find and sketch the domain of this function.

b.) Compute the partial derivatives  $f_x$  and  $f_y$ .

c.) Show that  $f(x, y)$  satisfies the partial differential equation  $f_{xx} = f_{yy}$ .