SIAM REVIEW
Vol. 47, No. 3, pp. 579–626

Book Reviews

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Working Analysis. *By Jeffery Cooper.* Elsevier Science & Technology, Amsterdam, 2004. $99.95. xvi+663 pp., hardcover. ISBN 0-12-187604-7.

Mathematical Analysis (a.k.a. introductory analysis, advanced calculus, introductory real analysis, etc.) is considered to be a foundational course for math majors; it is intended to initiate the students into the world of rigorous proofs, teaching them how to understand proofs and how to do them. Instead of going into the history of this very traditional course starting from Goursat and Vallée-Poussin, I would like to relate a brief personal story.

Mathematical analysis was one of the first courses that the leaders of our department at the time, Bob O'Malley and Jim Voytuk, entrusted me with. By that time I had already digested the news that in America calculus is taught twice, first without proofs and after that with proofs, and I had somewhat ambiguous feelings about this separation. On one hand, since I came from one of the last countries in the world where calculus was still taught in one sweep, such duplication seemed a bit wasteful. On the other hand, I used to have plenty of friends, chemists and engineers, for whom that one and only calculus course was close to the scariest experience of their lives; to pass they were forced to memorize proofs, sometimes without understanding a single word of them.

One of the reasons I was excited at the prospective of teaching mathematical analysis was that it was usually taught from Rudin's book [1]. In Russia the standard university text for calculus had been the three-volume mammoth by G. M. Fichtengolz (2,272 pages, and as Dave Barry likes to say, "I am not making it up"; almost no pictures either, mind you) or a two-volume subset of it (just a meager 1,500 pages; the page counts are by the 2003 Russian edition of the books that are still going strong, 45 years after the author's death). However, at my alma mater, Moscow State, the textbook was Rudin fortified by a healthy portion of exercises from *Problems and Exercises of Mathematical Analysis* by Demidovich. With all the boring and complicated differentiations, substitutions, and other goodies of "regular" calculus out of the way, I was really looking forward to conveying the beauty and elegance of the book and the subject to the students. Little did I know....

When I collected the first homework assignment it clearly confirmed the suspicious signs that I tried to ignore before. As it turned out, not only did the students experience huge difficulties with the simplest notions of rigorous calculus, but also the whole idea of a rigorous proof was somewhat foreign to most of them (as was their teacher, who definitely had a weird accent). Rudin's book was of little help: its elegant proofs were sometimes opaque in the extreme. It appeared that whenever several proofs were available, the shortest was invariably selected, no matter how tricky it was. I have a simple theory concerning the laconic style of the book. In the late 1940s, when Rudin wrote the book, inserting formulas into the typescript was done by hand, and for people with less than calligraphic handwriting at the price of tremendous pain. Of course senior professors mercilessly exploited their students and managed to write long books, but Rudin, being a junior professor, probably had to do this work himself.

Over the years I taught this course six times, each of them from different textbooks, and it was not for the love of reading. Books ranged in price from \$12.95 to \$132.99, and in approach from the book whose author will remain unnamed, in which mathematical analysis looked almost like a branch of algebra (with chains of sections, subsubsections, definitions, and propositions with no motivation whatsoever) to the beautiful, well-thought, and very geometrical book by Strichartz [2] (my favorite). In my opinion the book by Cooper is a viable competitor to Strichartz. It's too bad that I got it in November; had I gotten it early enough, I would have definitely tried it out in class.

Briefly, what in my opinion is wrong with mathematical analysis as we usually teach it and, correspondingly, with many textbooks? The surgery that separated mathematical analysis from calculus left the former with no applications which are not of a "conceptual" kind (i.e., leading to some rigorous mathematical results). That puts the students who are not exceptionally gifted mathematically into a very hopeless situation. The whole subject becomes a difficult and pointless game for them: prove theorems to prove other theorems. By the way, I don't think the aforementioned surgery left calculus unscathed either. There is a rather common belief that engineers and scientists should learn calculus because of their need to use its technical tools and language. Be it so, I think it is no less beneficial for them to acquire some basics of a mathematical way of thinking, in particular a critical and rigorous approach to problem formulation and solving among other things. A good proof or two here and there is probably more applicable than the ability to find the second derivative of an implicitly given function or writing a cardioid in polar coordinates (with negative r at that).

In *Working Analysis* Jeffery Cooper is determined to bring back the soul of analysis, which is to solve real problems; to the list of "usual suspects" Cooper adds nontrivial problems from economics, sociology, and numerical analysis. The book is divided into two parts, which roughly correspond to two semesters. The first part covers the usual grounds: sequences, continuity, the derivative, Taylor polynomials, integration, and series, all of them on the real line. In addition there is a nice little chapter on solving equations in one-dimension that discusses fixed-point iteration and Newton's method. Many examples and illustrations of calculus results are drawn from numerical mathematics and, as a free benefit, are very instructive and useful per se. Some of the examples are followed by numerical results that students can easily reproduce. Take, for instance, the golden-section search algorithm. There is also a clear intent to select the proofs that contain some constructive and usable computational ideas.

I understand that a book on "working" analysis would necessarily exclude some more pure concepts. On the other hand, the book is a text for math majors, and things like countable and uncountable sets, Cantor's diagonalization process, and Cantor sets are extremely attractive for beginners. We may disagree on whether these notions belong to working analysis, but I don't think it is fair to deprive students of the pleasure of encountering them in the analysis course. Where else will they learn that there are many more irrational numbers than rational ones?

To continue with my nit-picking, it is a wonderful idea to have a separate section, "Using Inequalities," that explains how to deal with inequalities, a subject many students are not too good at. On the other hand, $\varepsilon - N$ or $\varepsilon - \delta$ type definitions and arguments are notoriously difficult for students; for example, I have seen several "proofs" that started with the words "let $\varepsilon = 1$." I believe that Cooper overestimates students' mathematical maturity when he devotes just a few lines to these and some other important notions.

Here I would like to intersperse another personal recollection. I remember how I. M. Gelfand explained why he was a "superb teacher" (his own words, which I agree with 100%). In the Soviet Union in the beginning of 1930s, the government encouraged a huge influx of ill-educated workers, many without even a high-school education, into the universities. I cannot dwell on the causes of this phenomenon, which were mostly political. To cater to these students, universities introduced special departments, *Rabfacs* (in English it would be translated as *Wordeps*, from "Workers' Departments"). Gelfand, who at the time was 17–20 years old, taught trigonometry at one of those *Rabfacs*. He started his class by writing an α on the board and saying "It is a Greek letter α, it looks like an a, but it is a little different, it is called alpha. Mathematicians want to be fancy and like to call angles by Greek letters; it is not a big deal, you will get used to this." Without this introduction many students would have been lost right from the first minute. I have the feeling that Cooper could do a better job in explaining his "alphas."

The second part of the book, while containing some traditional topics (such as topology and differentiation in R^n, the implicit function theorem, and integration and change of variables in multiple integrals), also includes two extensive chapters on optimization. The book is completed by an applications chapter that, among other topics, contains sections on mollification and convolution and a nice discussion of Euler's equations for an inviscid fluid that are mostly treated as an example of change of variables in integrals. In the chapters on solving systems of equations and on constrained optimization the level of sophistication picks up substantially. It is feasible to design a good senior-level course on optimization based on this part of the book.

To summarize, this textbook is based on a very healthy philosophy that it is easier to learn mathematical analysis when it is intertwined with meaningful applications. The book is fun to read and, I am sure, will be fun to learn from. It is less polished than [1] or [2], and I hope it will have a long and fruitful life so that the author will have a chance to make it even better in the future editions.

REFERENCES

[1] W. RUDIN, *Principles of Mathematical Analysis*, 3rd ed., McGraw-Hill, New York, 1976.

[2] R. S. STRICHARTZ, *The Way of Analysis*, Jones and Bartlett, Boston, 2000.

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