

Time allowed 50 minutes

Please answer all 4 questions, showing your work in resonable detail.

One sheet of notes may be used for the test.

Books, papers and **calculators** are not permitted.

1. (13 pts) Solve the initial value problem:

$$x \frac{dy}{dx} - y = x^3, \quad y(1) = 0.$$

2. (12 pts.) Find the explicit solution $y = \phi(t)$ of the initial value problem

$$\frac{dy}{dt} = \frac{1 - 2t}{y}, \quad y(1) = -2,$$

and find the interval where the solution is valid.

- 3.(a) (9 pts) A tank initially contains 200 gallons of water and 200 grams of salt. Water containing 3 g/gal of salt flows into the tank at a rate of 2 gal per minute. The mixture in the tank also flows out of the tank at the same rate of 2 gal/min. Set up an initial value problem whose solution is the amount of salt in the tank at time t . **Do not solve the problem.**

(b) (3 pts. *BONUS*) What will be the amount of salt in the tank asymptotically as $t \rightarrow \infty$?

(c) (3 pts.) The radioactive isotope plutonium-241 decays exponentially, $Q(t) = Q_0 e^{-0.0525t}$, where Q_0 is the initial amount and t is measured in years. Determine the half-life time of Pu-241. (It is OK to leave your answer in terms of logarithms).

4. (a) (10 pts.) Consider the equation

$$\frac{dy}{dt} = y(y - 2)(5 - y)$$

Find the equilibrium solutions. If the initial value is $y(0) = 1$, sketch the solution $y(t)$, find $\lim_{t \rightarrow \infty} y(t)$, the same question for $y(0) = 4$. Explain your answers.

(b) (3pts.) Given the differential equation $dy/dt = f(t, y)$. In your own words, explain what integral curves of the eqation are (the geometrical meaning of solutions).

Prob.#	Pts.
1	
2	
3	
4	
Total	