

# Homework Assignments

## Introduction to Number Theory, Spring 2011

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# 1 Homework 1: Due Thursday, February 3rd, in class

The problems below should be done using only the results in section 1.2. That means that you cannot use the result about the unique factorization into prime numbers and get full credit. Please feel free to contact me or the TA during office hours for help with problems 1-7. Problem 8 may possibly be somewhat harder and no hints will be given. Problems such as number 8 will occur on each problem set. If you don't get these problems, you can compensate in your grade by taking advantage of the in-class extra credit.

1. Section 1.2, problem 1b: Write out the calculations as in example 1
2. Section 1.2, problem 2: Write out the calculations as in examples 1,2 and then make a table as in example 3
3. Section 1.2, problem 3c,d: Just make a table as in example 3
4. Section 1.2, problem 20
5. Section 1.2, problem 26
6. Section 1.2, problem 34
7. Section 1.2, problem 43
8. Section 1.2, problem 46 (This may be a bit more challenging. No Hints or help will be given.)

## 2 Homework 2: Due Thursday, February 10th, in class

The problems below should be done using only the results in sections 1.2,1.3,1.4. Please feel free to contact me or the TA during office hours for help with problems 1-7. Problem 8 may possibly be somewhat harder and no hints will be given. Problems such as number 8 will occur on each problem set. If you don't get these problems, you can compensate in your grade by taking advantage of the in-class extra credit iclicker questions.

1. Section 1.3, problem 6
2. Section 1.3, problem 7
3. Section 1.3, problem 10
4. Section 1.3, problem 24
5. Section 1.3, problem 26
6. Section 1.3, problem 27
7. Section 1.4, problem 2
8. Section 1.3, problem 41 (This may be a bit more challenging. No Hints or help will be given.)

### 3 Homework 3: Due Thursday, February 17th, in class

Please feel free to contact me or the TA during office hours for help with problems 1-7. Problem 8 may possibly be somewhat harder and no hints will be given. Problems such as number 8 will occur on each problem set. If you don't get these problems, you can compensate in your grade by taking advantage of the in-class extra credit iclicker questions.

1. Section 2.1, problem 2
2. Section 2.1, problem 6
3. Section 2.1, problem 13
4. Section 2.1, problem 20
5. Section 2.1, problem 32
6. Section 2.1, problem 34
7. Section 2.1, problem 43
8. Section 2.1, problem 44 (This may be a bit more challenging. No Hints or help will be given.)

## 4 Homework 4: Due Thursday, February 24th, in class

Please review the academic integrity policy on the opening day handout linked to the course web page. Please note that asking for help on the internet, or reading a response to a request for help on the internet is not permitted.

Please feel free to contact me or the TA during office hours for help with problems 1-7. I'm also available by email or by appointment.

Problem 8 may possibly be somewhat harder and no hints will be given. Problems such as number 8 will occur on each problem set. If you don't get this problem, you can compensate in your grade by taking advantage of the in-class extra credit iclicker questions.

Here are some helpful tips in writing homework solutions:

1. Use Theorems and definitions from the book. You should look for these first. It is helpful to memorize them in advance.
2. Use formulas and notation where appropriate. In some cases, you need to introduce notation to make things precise. When introducing new symbols, or variables, you need to say what they are. (e.g. Assume  $p_1, p_2, \dots, p_k$  are the prime factors of  $n$ )
3. Do not only use formulas, also use sentences. You need to balance formulas and sentences.
4. When there is more than one case, please do all cases. Do not claim other cases are similar without asking me if it is ok.
5. Writing things such as  $x = a/b$  or  $y = \sqrt{n}$  is not permitted in this class without further comment or conditions because  $x$  and  $y$  are assumed to be integers. Either state that  $x$  and  $y$  are real number or explain why they are integers (unless it is obvious in formulas like  $x = a/(a, m)$ .)

Be sure to show all work on computational problems.

1. Section 2.2, problem 5c and 5d
2. Section 2.2, problem 6a,6b,6c
3. Section 2.2, problem 8
4. Section 2.3, problem 3
5. Section 2.3, problem 6
6. Section 2.3, problem 20
7. Section 2.3, problem 27
8. Section 2.3, problem 38 (This may be a bit more challenging. No Hints or help will be given.)

## 5 Homework 5: Due Thursday, March 31st, in class

Please review the academic integrity policy on the opening day handout linked to the course web page. Please note that asking for help on the internet, or reading a response to a request for help on the internet is not permitted.

Please feel free to contact me or the TA during office hours for help with problems 1-7. I'm also available by email or by appointment.

Problem 8 may possibly require more programming skills and no programming support will be given. Problems such as number 8 will occur on each problem set. If you don't get this problem, you can compensate in your grade by taking advantage of the in-class extra credit iclicker questions.

Here are some helpful tips in writing homework solutions:

1. Use Theorems and definitions from the book. You should look for these first. It is helpful to memorize them in advance.
2. Use formulas and notation where appropriate. In some cases, you need to introduce notation to make things precise. When introducing new symbols, or variables, you need to say what they are. (e.g. Assume  $p_1, p_2, \dots, p_k$  are the prime factors of  $n$ )
3. Do not only use formulas, also use sentences. You need to balance formulas and sentences.
4. When there is more than one case, please do all cases. Do not claim other cases are similar without asking me if it is ok.
5. Writing things such as  $x = a/b$  or  $y = \sqrt{n}$  is not permitted in this class without further comment or conditions because  $x$  and  $y$  are assumed to be integers. Either state that  $x$  and  $y$  are real number or explain why they are integers (unless it is obvious in formulas like  $x = a/(a, m)$ .)

Be sure to show all work on computational problems.

1. Section 2.3, problem 15 (Solve this using the techniques in section 2.3. See Theorem 2.20 and surrounding discussion.)
2. Section 2.6, problem 4 (Solve this using the techniques of section 2.6)
3. Section 2.6, problem 5 (Solve this using the techniques of section 2.6)
4. Section 2.7, problem 1
5. Section 2.7, problem 2
6. Section 2.7, problem 4
7. Section 2.7, problem 6 (Note that  $p$  is prime,  $m$  is not prime.)
8. Write a program (in a suitable language) that tests to see if a number is a strong pseudoprime to the base  $b$ . Use this program to test if a collection of random numbers (any random or pseudorandom number generator is fine) that are each about 5-10 digits long to see if they are  $spsp(2)$ .
  - (a) How many random numbers do you test before you get 5 numbers that are  $spsp(2)$ ?
  - (b) How many of the 5 numbers that you got in part a are  $spsp(3)$ ?
  - (c) How many of the numbers that you got in part b are  $spsp(7)$ ?

Summarize your findings.

(Partial credit for using an existing strong pseudoprime tester to answer the questions.)

## 6 Homework 6: Due Thursday, April 7th, in class

Please review the academic integrity policy on the opening day handout linked to the course web page. Please note that asking for help on the internet, or reading a response to a request for help on the internet is not permitted.

Please feel free to contact me or the TA during office hours for help with problems 1-7. I'm also available by email or by appointment.

Problem 8 may be a bit harder. If you don't get this problem, you can compensate in your grade by taking advantage of the in-class extra credit iclicker questions.

Here are some helpful tips in writing homework solutions:

1. Use Theorems and definitions from the book. You should look for these first. It is helpful to memorize them in advance.
2. Use formulas and notation where appropriate. In some cases, you need to introduce notation to make things precise. When introducing new symbols, or variables, you need to say what they are. (e.g. Assume  $p_1, p_2, \dots, p_k$  are the prime factors of  $n$ )
3. Do not only use formulas, also use sentences. You need to balance formulas and sentences.
4. When there is more than one case, please do all cases. Do not claim other cases are similar without asking me if it is ok.
5. Writing things such as  $x = a/b$  or  $y = \sqrt{n}$  is not permitted in this class without further comment or conditions because  $x$  and  $y$  are assumed to be integers. Either state that  $x$  and  $y$  are real number or explain why they are integers (unless it is obvious in formulas like  $x = a/(a, m)$ .)

Be sure to show all work on computational problems.

1. Section 2.8, problem 6
2. Section 2.8, problem 8 (Show all work)
3. Section 2.8, problem 10 (Use a table of powers of 3 modulo 17 to apply the important statement at the bottom of page 100, For a primitive root  $g$  modulo  $m$ , " $g^i \equiv g^j \pmod{m}$ " iff  $i \equiv j \pmod{\phi(m)}$ .)
4. Section 2.8, problem 14
5. Section 2.8, problem 18
6. Section 2.8, problem 21
7. Section 2.8, problem 22
8. Section 2.8, problem 29 (This may be a bit more challenging. No Hints or help will be given.)

## 7 Homework 7: Due Thursday, April 28th, in class

Please review the academic integrity policy on the opening day handout linked to the course web page. Please note that asking for help on the internet, or reading a response to a request for help on the internet is not permitted.

Please feel free to contact me or the TA during office hours for help with problems 1-8. I'm also available by email or by appointment.

Here are some helpful tips in writing homework solutions:

1. Use Theorems and definitions from the book. You should look for these first. It is helpful to memorize them in advance.
2. Use formulas and notation where appropriate. In some cases, you need to introduce notation to make things precise. When introducing new symbols, or variables, you need to say what they are. (e.g. Assume  $p_1, p_2, \dots, p_k$  are the prime factors of  $n$ )
3. Do not only use formulas, also use sentences. You need to balance formulas and sentences.
4. When there is more than one case, please do all cases. Do not claim other cases are similar without asking me if it is ok.
5. Writing things such as  $x = a/b$  or  $y = \sqrt{n}$  is not permitted in this class without further comment or conditions because  $x$  and  $y$  are assumed to be integers. Either state that  $x$  and  $y$  are real number or explain why they are integers (unless it is obvious in formulas like  $x = a/(a, m)$ .)

Be sure to show all work on computational problems.

1. Section 4.2, problem 1
2. Section 4.2, problem 5
3. Section 4.2, problem 6
4. Section 4.2, problem 8
5. Section 4.2, problem 10
6. Section 4.2, problem 11
7. Section 4.2, problem 16
8. Section 4.2, problem 20

## 8 Homework 8: Due Monday, May 9th, in class

Please review the academic integrity policy on the opening day handout linked to the course web page. Please note that asking for help on the internet, or reading a response to a request for help on the internet is not permitted.

Please feel free to contact me or the TA during office hours for help with problems 1-7. I'm also available by email or by appointment.

Problem 8 may be a bit harder. If you don't get this problem, you can compensate in your grade by taking advantage of the in-class extra credit iclicker questions.

Here are some helpful tips in writing homework solutions:

1. Use Theorems and definitions from the book. You should look for these first. It is helpful to memorize them in advance.
  2. Use formulas and notation where appropriate. In some cases, you need to introduce notation to make things precise. When introducing new symbols, or variables, you need to say what they are. (e.g. Assume  $p_1, p_2, \dots, p_k$  are the prime factors of  $n$ )
  3. Do not only use formulas, also use sentences. You need to balance formulas and sentences.
  4. When there is more than one case, please do all cases. Do not claim other cases are similar without asking me if it is ok.
  5. Writing things such as  $x = a/b$  or  $y = \sqrt{n}$  is not permitted in this class without further comment or conditions because  $x$  and  $y$  are assumed to be integers. Either state that  $x$  and  $y$  are real number or explain why they are integers (unless it is obvious in formulas like  $x = a/(a, m)$ .)
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1. Find all Pythagorean Triples  $x, y, z$  (primitive and non-primitive) such that  $z \leq 20$
  2. Find a formula in terms of 1 parameter that gives all Primitive Pythagorean Triples  $x, y, z$  such that  $z = y + 1$ . Prove your result. (Your formula should give only Primitive Pythagorean Triples of this type for positive integer parameter values.)
  3. Find a formula in terms of 1 parameter that gives all Primitive Pythagorean Triples  $x, y, z$  such that  $z = x + 2$ . Prove your result. (Your formula should give only Primitive Pythagorean Triples of this type for positive integer parameter values.)
  4. Are there any Primitive Pythagorean Triples  $x, y, z$  so that  $z = x + 4$ ? Demonstrate one or prove there are none.
  5. Find a formula that gives all Pythagorean Triples  $x, y, z$  so that  $z = x + 4$ .
  6. Show that if  $x, y, z$  is a Pythagorean Triple then either  $x$  or  $y$  is divisible by 3
  7. Show that if  $x, y, z$  is a Pythagorean Triple then either  $x$  or  $y$  or  $z$  is divisible by 5
  8. Find (in terms of  $n$ ) a simple formula and/or a good simple approximation for the number of rational points  $(x/z, y/z)$  that lie on the unit circle that have  $z \leq n$ .